Concentration effect on equilibrium fractionation of Mg-Ca isotopes in carbonate minerals: Insights from first-principles calculations

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Abstract

Naturally occurring carbonates have a wide variation in Mg and Ca contents. Using the density-functional-theory calculations, this study examines the effect of Mg and Ca concentrations on bond lengths and equilibrium fractionation factors of Mg-Ca isotopes among calcite-type carbonate minerals (Mg\(_x\)Ca\(_{1-x}\)CO\(_3\)). Mg content \(x\) and Ca content \((1-x)\) of the investigated carbonate minerals range from \(1/12\) to \(1\) and from \(1/36\) to \(1\), respectively. Concentration of Ca and Mg in carbonates have significant effects on Ca–O and Mg–O bond lengths when \(x\) is close to 0, 0.5 or 1. Because equilibrium isotope fractionation factors \((10^3 \ln a)\) are mainly controlled by their relevant bond strengths, which can be measured using their average bond lengths, \(10^3 \ln a\) of \(^{26}\)Mg/\(^{24}\)Mg and \(^{44}\)Ca/\(^{40}\)Ca between calcite-type carbonate minerals and dolomite also vary dramatically with Mg content, especially when \(x\) is close to 0 and 1. For instance, at 300 K, \(10^3 \ln a\) of \(^{26}\)Mg/\(^{24}\)Mg between Mg\(_{1/12}\)Ca\(_{11/12}\)CO\(_3\) and dolomite \((x = 0.5)\) is \(-4.3\)‰, while \(10^3 \ln a\) of \(^{44}\)Ca/\(^{40}\)Ca between Mg\(_{23/24}\)Ca\(_{1/24}\)CO\(_3\) and dolomite is \(-6\)‰. Dolomite is enriched in \(^{26}\)Mg but depleted in \(^{44}\)Ca relative to all other carbonate minerals, which is consistent with it having the shortest Mg–O bond length and the longest Ca–O bond lengths among all carbonates. At 300 K, a small change of \(x\) from 0.5 to 0.6 in dolomite could result in 1‰ variation in \(10^3 \ln b\) of \(^{26}\)Mg/\(^{24}\)Mg. Therefore, the concentration effect in carbonate minerals should be taken into account when applying the isotope fractionation factors of carbonate minerals to understand geochemical processes.

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Keywords: First-principle calculations; Concentration effect; Equilibrium fractionation; Mg-Ca isotopes; Carbonates

1. INTRODUCTION

Carbonates are important components of sediments, recording many fundamental environmental and geochemical processes. Calcite, dolomite, and magnesite are the major carbonate minerals in sedimentary rocks. C, O, Mg, and Ca stable isotopic compositions of these carbon-
ates have a number of essential implications. For example, the oxygen isotopic compositions in carbonate minerals have been used as a thermometry investigating paleoclimate change (e.g., Kim and O’Neil, 1997; Benis et al., 1998; McDermott, 2004; Lea, 2014). Carbon isotope excursions in marine carbonates and sedimentary organic matters are often related to catastrophic events in the Earth’s history (e.g., Kaufman and Knoll, 1995; Hoffman et al., 1998; Shields-Zhou and Oeh, 2011; Johnston et al., 2012; Sahoo et al., 2012), and they are the key parameters in the study of carbon cycling and paleo-climate changes (Werne and Hollander, 2004; Ader et al., 2009; Meyer et al., 2013).

More recently, Ca isotopic compositions in marine carbonates have been used to constrain global Ca cycling (De La Rocha, 2000; Heuser et al., 2005; Farkas ˇ et al., 2007), and Mg isotope data in carbonates have been applied to constrain the oceanic cycling of Mg and continental weathering (Tipper et al., 2006b; Higgins and Schrag, 2010; Li et al., 2015). Furthermore, since Mg and Ca fluxes are usually coupled with carbon cycles, Mg-Ca isotopes have great potential to trace deep carbon recycling (DePaolo, 2004; Huang et al., 2011, 2015; Yang et al., 2012; Zhang and Li, 2012; Cheng et al., 2015; Liu et al., 2015). Reported δ26/24Mg values of precipitated marine carbonates vary from around −5‰ to −1‰ in magnesian calcites (Galy et al., 2002; de Villiers et al., 2005; Tipper et al., 2006a,b; Buhl et al., 2007; Hippler et al., 2009), and from −2‰ to −1‰ in dolomites (Galy et al., 2002; Chang et al., 2003; Jacobson et al., 2010). It is not clear whether the δ26/24Mg variations in carbonate minerals reflect different precipitation environments or Mg isotopic fractionation among Mg-bearing species. Therefore, in order to better apply stable isotope data into geochemical studies, it is critical to understand equilibrium isotope fractionation factors among carbonate minerals.

Chemical compositions of carbonate minerals vary tremendously as solid solutions due to the incorporations of divalent cations including Mg2+, Sr2+, Ba2+, and Ca2+ (Tesoriero and Pankow, 1996; Huang and Fairchild, 2001). Generally, Mg2+ cations could substitute Ca2+ cations to form low-Mg calcite (with <4 mol% Mg, mole ratio hereafter defined as Mg/(Mg + Ca)) and high-Mg calcite (with >4 mol% Mg). The Mg content of experimentally synthesized calcites in aqueous solutions could be up to 22 mol% (Mucci and Morse, 1983; Mucci, 1986; Huang and Fairchild, 2001; Meldrum and Hyde, 2001; Li et al., 2012), but magnesiums with low Ca content were rarely reported in literature. Although naturally occurring dolomites generally contain 50 mol% MgCO3 in an idealized mineral structure, they also show variable Mg contents (about 40–50 mol%) (Drits et al., 2005).

Variations of the chemical compositions may affect structures of minerals, which further impacts the equilibrium isotope fractionation factors among minerals. For example, based on first-principles calculations, Feng et al. (2014) reported that equilibrium Ca isotope fractionation factors between clinopyroxene (cpx) and orthopyroxene (opx) are controlled not only by temperature, but also the Ca content in opx. Pinilla et al. (2015) reported the noticeable equilibrium Mg isotopes fractionation between Mg-calcites with Mg contents of 3.12 mol% and 6.25 mol%. However, their calculations were conducted with the fixed cell parameters, which means that these two investigated Mg-calcites may not be under the same pressure and thus they are not in thermodynamic equilibrium. It is therefore important to scrutinize the possible concentration effect on stable isotope fractionation among carbonate minerals.

It is challenging to synthesize carbonate minerals with a wide range of controllable Mg contents at ambient conditions through inorganic experiments, and synthesis of dolomite is even referred to as the “dolomite problem” for decades (Zenger et al., 1980; Land, 1998; Warren, 2000). Therefore, experimental determination of the equilibrium isotopes fractionation between carbonate minerals with variable chemical compositions remains a great challenge. No experimental study has yet been reported to examine the concentration effect on the equilibrium isotope fractionation between carbonate minerals. Mg isotope fractionation factors between carbonate minerals (Mg-calcite and dolomite) and aqueous Mg solutions were experimentally measured at various temperatures (Pearce et al., 2012; Li et al., 2012, 2015; Saulnier et al., 2012; Mavromatis et al., 2013) and theoretically calculated (Rustad et al., 2010; Schauble, 2011), but there is a significant discrepancy between experimental and theoretical results. This highlights the complexity of determining isotope fractionation factors, such as kinetic effect (Saenger and Wang, 2014). The Mg concentration variations in synthesized carbonate minerals may also contribute to the complexity.

Recent advances in computational capabilities prove that the density functional theory (DFT) provides a powerful tool to calculate equilibrium isotope fractionation factors. DFT calculations, which show comparable precision relative to the well-designed experiments (Lejaeghere et al., 2016), have been widely used to calculate equilibrium fractionation factors for a number of systems such as Mg, Si, Ca, and V (Schauble et al., 2004; Griffith et al., 2008; Méheut et al., 2009; Schauble, 2011; Li and Liu, 2011; Kowalski and Jahn, 2011; Kowalski et al., 2013; Huang et al., 2013, 2014; Feng et al., 2014; Wu et al., 2015). Natural calcite and magnesite are solid solutions with substantial variations in Ca-Mg concentration. For better understanding the concentration effect on equilibrium isotope fractionation, we estimate fractionation factors of Mg and Ca isotopes among carbonate minerals (Mg, Ca1−x,−xCO3) with x varying from 0 to 1 using the DFT calculations. We focus on the effect of Ca-Mg substitutions on carbonate mineral structures, Ca-O and Mg-O bonding, and equilibrium isotope fractionation. Our results reveal a significant concentration effect on equilibrium fractionation of Mg-Ca isotopes in carbonate minerals.

2. CALCULATION METHODS

Equilibrium isotope fractionation arises from changes in vibrational frequencies caused by isotopic substitution of an interested element in two phases (Bigeleisen and Mayer, 1947; Urey, 1947). The isotope fractionation factor of element X between two phases A and B, $\alpha_{A,B}$, is the ratio...
of their isotope ratios of these two phases under equilibrium. According to Richet et al. (1977), the isotope fractionation factor between Phase A and an ideal gas of X atoms is the reduced partition function ratio \( \beta_A \) of the Element X, defined in the harmonic approximation as:

\[
\beta_A = \frac{Q_A}{Q_l} = \prod_{i} \frac{u_{ih} e^{-\omega_i h}}{u_{il} e^{-\omega_i l}} \frac{1 - e^{-\omega_i t}}{1 - e^{-\omega_i l}}
\]  

(1)

where \( Q \) is the vibrational partition function, index \( h \) and \( l \) refer to the heavy and light isotopes, respectively; \( i \) is a running index of vibrational frequency mode, and \( N \) is the number of atoms in the unit cell. A crystal with \( N \) atoms has \( 3N \) vibrational modes and the product runs over all \( 3N \) phonon modes. \( u_{ih} \) and \( u_{il} \) are defined as:

\[
u_{ih} \text{ or } \nu_{il} = h\omega_{ih} \text{ or } \nu_{il}/k_BT
\]  

(2)

where \( h \) is the Planck constant, \( k_B \) is the Boltzmann constant, \( T \) is temperature in Kelvin, and \( \omega_{ih} \text{ or } \nu_{il} \) is the vibrational frequency of the \( \nu \)th mode. Following Richet et al. (1977), the equilibrium isotope fractionation between two Phases A and B can be rewritten in per mil as:

\[
\Delta x_{A-B} = 10^3 \ln x_{A,B} = 10^3 \ln \beta_A - 10^3 \ln \beta_B
\]  

(3)

The calculation details in this study are similar to that published in previous work (Schauble, 2011; Li and Liu, 2011; Huang et al., 2013, 2014; Feng et al., 2014; Wu et al., 2015). All calculations were performed using an open-source software “Quantum Espresso” based on the DFT, plane wave, and pseudopotential (Giannozzi et al., 2009). Electronic wave functions are expanded by a plane-wave basis set. The interaction between the valence electrons and the ionic core is described using pseudopotential. Local density approximation (LDA) for exchange correlation functional (Perdew and Zunger, 1981) was used in this study because of its advantages on calculating mineral structures and thermodynamic properties of minerals (Wentzcovitch et al., 2010; Huang et al., 2013). The plane-wave cutoff energy is 70 Ry.

The pseudopotentials of calcium and carbon were generated using Vanderbilt method (Vanderbilt, 1990) with a configuration of \( 3s^23p^44s^1 \) and a 1.85 Bohr cutoff radius for calcium, and a configuration of \( 2s^22p^5 \) and a 1.3 Bohr cutoff radius for carbon. The pseudopotential of magnesium was generated using the method of von Barth and Car for all channels using a 2.5 Bohr cutoff radius and five configurations, \( 3s^23p^4, 3s^23p^23d^2, 3s^23p^33d^0,5, 3s^23p^35, 3s^23d^1 \), with weights of 1.5, 0.6, 0.3, 0.3, 0.2, respectively. The oxygen pseudopotential was generated by the method of Troullier and Martins (1991) with configuration \( 2s^22p^4 \) and a cutoff radius of 1.45 Bohr. Brillouin zone integrations over electronic states were performed with \( N_1 \times N_2 \times N_3 \) k-point grid dependent on the size of unit cells (see Table S1).

The initial calcite crystal structures were obtained from previous experimental studies (Graf, 1961; Markgraf and Reeder, 1985). Crystal structures with variable Mg contents were well optimized using variable cell shape molecular dynamics (Wentzcovitch, 1991) and the residual forces converge within \( 10^{-8}\) Ry/Bohr. The dynamical matrices were calculated on a regular q mesh dependent on the size of unit cells (Table S1) using the density-functional perturbation theory (DFPT) and then interpolated on a dense q mesh to obtain the vibrational density of the state of minerals.

## 3. RESULTS

### 3.1. Relaxed crystal structures

The unit structure cell of calcite with space group R-3c contains six symmetry-equivalent Ca atoms (Graf, 1961; Markgraf and Reeder, 1985). All initial carbonate structures before optimization, with Mg/(Mg + Ca) (mole ratio hereafter) varying from 1/12 to 1, were obtained by replacing Ca atoms with Mg atoms in the primitive calcite cell. For instance, one of the 12 Ca atoms in a 60-atom cell of calcite was substituted by one Mg atom to make a Mg/(Mg + Ca) of 1/12. Because the initial carbonate structures with Mg/(Mg + Ca) of 2/6, 3/6, and 4/6 have more than one configurations, all nonequivalent configurations in a 30-atom cell were calculated and the configurations with the lowest total energy were selected for dynamical matrices calculations. To test the effect of the supercell size, we also consider the structure with Mg/(Mg + Ca) of 2/12 in a 60-atom supercell, which is twice as large as that with Mg/(Mg + Ca) of 1/6 in a 30-atom supercell. The configuration with the lowest total energy among all nonequivalent configurations was selected as for dynamical matrices calculations. The optimized crystal structures of carbonates with variable Mg/(Mg + Ca) are shown in Fig. 1 with emphasis on Mg–O and Ca–O polyhedrons. Although the initial structures of carbonates with Mg/(Mg + Ca) of 3/6 and 1 are calcite-type structures, their optimized structures are consistent with dolomite and magnesite structures, respectively, as indicated by the fractional coordinates shown in Table S2. Both Ca and Mg have a coordination number (CN) of six in all calcite-type carbonates with variable Mg/(Mg + Ca), while Ca is ninefold coordinated in aragonite.

The calculated crystal lattice parameters of all carbonate minerals (Table S3), are consistent with experimental results within 1% (Fig. 2) after considering the temperature effect estimated using the volume expansion of calcite from static condition to 300 K, in which the volume of calcite increases by 1.8%. After the temperature correction, the crystal volumes are consistent with experimental data within 1%, indicating that the predicted crystal volumes in this study are reliable. The calculated vibrational frequencies of calcite, dolomite, magnesite, and aragonite agree well with experimental results (Fig. 3 and Table S4), with the slope between calculated frequencies and measured frequencies being 0.998 \pm 0.020 (1\sigma, \text{R}^2 = 0.9998) (Fig. 3 and Table S4). This demonstrates that the calculated equilibrium isotope fractionation factors are reliable and accurate. Therefore, the LDA is a good exchange correlation functional for calculations of isotope fractionation among minerals, as demonstrated by previous studies (e.g., Li et al., 2011; Huang et al., 2013; Feng et al., 2014; Wu et al., 2015). Based on the analysis in previous work (Méheut et al., 2009), a deviation of n% on phonon frequencies would induce an error of n% at low temperature and 2n% at high temperature on \( \ln \beta \). Therefore, the relat-
ative uncertainties of our calculated $10^3 \ln \beta$ and $10^3 \ln \sigma$ are estimated at about 4% and 6%, respectively, under investigated temperature intervals (Méheut et al., 2009; Kowalski et al., 2013; Feng et al., 2014).

### 3.2. Average Mg–O and Ca–O bond lengths in carbonates

Average Mg–O and Ca–O bond lengths in carbonate minerals with variable Mg/(Mg + Ca) at static condition are reported in Table 1 and Fig. 4. As shown in Fig. 4, average Mg–O and Ca–O bond lengths vary with Mg/(Mg + Ca) of carbonate minerals. Our results show that average Mg–O and Ca–O bond lengths of the carbonate mineral with Mg/(Mg + Ca) of 1/6 calculated from the 60-atom supercell are consistent to those obtained from the 30-atom system. Dolomite has the shortest average Mg–O bond length and the longest average Ca–O bond length among all calculated carbonate minerals (Fig. 4a and b). Because the size of Ca$^{2+}$ is larger than Mg$^{2+}$ in carbonates, we expect that the Mg–O bond in calcite is longer than that in magnesite. Therefore, the average Mg–O bond length increases in the order of dolomite < magnesite < calcite. For the same taken, when Ca occupies the small Mg site in magnesite, Ca–O bond in magnesite is shorter than the one in calcite and thus the sequence of average Ca–O bond length is dolomite > calcite > magnesite. The predicted dif-

![Fig. 1. Carbonate crystal structures with variable Mg/(Mg + Ca) with emphasis on Mg–O and Ca–O polyhedrons. Mg atoms are brown, Ca atoms cyan, C atoms dark brown, and O atoms red. All of these crystal structures are drawn using the software “VESTA” (Momma and Izumi, 2008). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image-url)
3.3. Isotope fractionation factors

The calculated reduced partition function ratios of $^{26}\text{Mg}/^{24}\text{Mg}$ ($10^3 \ln \beta_{^{26}\text{Mg}/^{24}\text{Mg}}$) and $^{44}\text{Ca}/^{40}\text{Ca}$ ($10^3 \ln \beta_{^{44}\text{Ca}/^{40}\text{Ca}}$) are shown in Fig. 5. The polynomial fitting parameters are reported in Table 2. $10^3 \ln \beta_{\text{minerals-dolomite}}$ (equilibrium isotope fractionation factors between Mg, Ca, and dolomite) of $^{26}\text{Mg}/^{24}\text{Mg}$ and $^{44}\text{Ca}/^{40}\text{Ca}$ are plotted as a function of temperature in Fig. 6. Relative polynomial fitting parameters are given in Table 3. Small differences between results of calcite-type carbonate minerals with Mg/(Mg + Ca) of 1/6 and 2/12 indicate that the effect of supercell size is secondary and negligible. $10^3 \ln \beta_{\text{Mg-dolomite}}$ increases in the order of Mg-calcite ($\text{Mg}_{1/12}\text{Ca}_{11/12}\text{CO}_3$) < magnesite < dolomite, which is generally consistent with previous theoretical results (Rustad et al., 2010; Schauble, 2011), as well as experimental and natural observations (Li et al., 2012, 2015; Saulnier et al., 2012; Mavromatis et al., 2013; Saenger and Wang, 2014).

As expected from the compositional dependence of average bond lengths on Mg/(Mg + Ca), $10^3 \ln \beta_{\text{Mg-dolomite}}$ and $10^3 \ln \beta_{\text{calcite-dolomite}}$ of calcite-type carbonate minerals vary significantly with Mg/(Mg + Ca) (Figs. 5–7). The more sensitive the bond lengths to Mg/(Mg + Ca), the more rapidly the reduced partition function ratios change with Mg/(Mg + Ca). The $10^3 \ln \beta_{\text{Mg-dolomite}}$ of dolomite (Mg/Ca = 1) is always higher than that of other calcite-type carbonate minerals with variable Mg proportions, while the $10^3 \ln \beta_{\text{calcite-dolomite}}$ of dolomite is always lower than that of other calcite-type carbonates and higher than that of aragonite (Fig. 6), consistent with the fact that dolomite has the shortest Mg–O and the longest Ca–O bond length among all carbonate minerals. Notably, isotope fractionation factors change dramatically with Mg/(Mg + Ca). For example, at 300 K, $10^3 \ln \beta_{\text{Mg-dolomite}}$ and dolomite range from $-4.3\%$ to 0 and $^{44}\text{Ca}/^{40}\text{Ca}$ from 0 to $<6\%$ when Mg/(Mg + Ca) varies from 1/12 to 1/2. Therefore, in addition to temperature, Mg/(Mg + Ca) in carbonate minerals also significantly affects equilibrium isotope fractionation.

4. DISCUSSION

4.1. Concentration effect on average Ca–O and Mg–O bond lengths in carbonates

Based on first-principles calculation, Feng et al. (2014) found that Ca–O bond length in opx is highly sensitive to its Ca concentration. Our results show that average Ca–O and Mg–O bond lengths in the carbonate minerals are also significantly affected by their Mg contents (Table 1 and Fig. 4), suggesting that the concentration effect on bond length should be a ubiquitous phenomenon in minerals. When Element A replaces Element B with a different radius, Element A should be adapted to the mineral structure controlled by Element B. The effect from Element B increases with decreasing concentration of Element A. However, when the concentration of Element A is low enough, the...
The equilibrium fractionation factors of Mg and Ca isotopes between calcite-type carbonate minerals (calcite-magnesite series) and dolomite vary significantly with Mg content in carbonate minerals. They are correlated with the average bond lengths with slightly different slopes for carbonate minerals with Mg/(Mg + Ca) < 1/2 and Mg/(Mg + Ca) > 1/2 (Fig. 8), which likely reflects the different dependences of average bond lengths on Mg/(Mg + Ca) due to the subtle disparity in structures between these two groups. When Mg/(Mg + Ca) < 1/2, Ca atoms are dominant relative to Mg atoms and show the major effect on structures. Similarly, Mg shows the major effect in carbonates when Mg/(Mg + Ca) > 1/2. Feng et al. (2014) also found that fractionation factors between opx and cpx are linearly correlated with average Ca–O bond lengths in cpx. Equilibrium isotope fractionations are controlled by relevant bond strengths (Bigeleisen and Mayer, 1947; Urey, 1947). As analyzed in previous studies (Urey, 1947; Schauble et al., 2004; Hill and Schauble, 2008; Young et al., 2009), shorter chemical bonds correspond to stronger bond strengths and higher vibrational frequencies, leading

4.2. Concentration effect on isotope fractionation factors

The equilibrium fractionation factors of Mg and Ca isotopes between calcite-type carbonate minerals (calcite-magnesite series) and dolomite vary significantly with Mg content in carbonate minerals. They are correlated with the average bond lengths with slightly different slopes for carbonate minerals with Mg/(Mg + Ca) < 1/2 and Mg/(Mg + Ca) > 1/2 (Fig. 8), which likely reflects the different dependences of average bond lengths on Mg/(Mg + Ca) due to the subtle disparity in structures between these two groups. When Mg/(Mg + Ca) < 1/2, Ca atoms are dominant relative to Mg atoms and show the major effect on structures. Similarly, Mg shows the major effect in carbonates when Mg/(Mg + Ca) > 1/2. Feng et al. (2014) also found that fractionation factors between opx and cpx are linearly correlated with average Ca–O bond lengths in cpx. Equilibrium isotope fractionations are controlled by relevant bond strengths (Bigeleisen and Mayer, 1947; Urey, 1947). As analyzed in previous studies (Urey, 1947; Schauble et al., 2004; Hill and Schauble, 2008; Young et al., 2009), shorter chemical bonds correspond to stronger bond strengths and higher vibrational frequencies, leading

![Fig. 4. Average (a) Mg–O bond and (b) Ca–O bond length of carbonate minerals versus Mg/(Mg + Ca).](image-url)
to enrichments in heavy isotopes relative to the longer and weaker chemical bonds. CN could also affect the bond strengths; however, the CNs of Ca and Mg in all calcite-type carbonates are six. Therefore, the bond lengths in calcite-type carbonate minerals dominantly control their bond strengths, resulting in a linear correlation between the inter-mineral isotope fractionations and the average bond lengths of carbonate minerals. This is similar to the case in opx and cpx where Ca is six-fold coordinated, and Ca isotopic fractionation between opx and cpx is also linearly correlated with Ca–O bond length in opx (Feng et al., 2014).

Pinilla et al. (2015) recently calculated equilibrium Mg isotopes fractionation factor between Mg-calcites with different Mg contents using the DFT method and they found that the calcite-type carbonate mineral with lesser Mg (3.12 mol%) is enriched in heavy Mg isotopes relative to the one with more Mg (6.25 mol%), which is in contrast with the expectation from this study that considers the carbonate minerals with Mg content higher than 6.25 mol %. The discrepancy probably arises from the difference in the calculation method. In their calculations, the cell parameters of carbonate minerals are fixed. Therefore, carbonate mineral with 3.12 mol% Mg must be under a higher pressure than the one with 6.25 mol% given that Mg atom is much smaller than Ca atom. Because increase of pressure can dramatically increase $10^3\ln \beta$ (Huang et al., 2013, 2014; Wu et al., 2015), this impractical pressure difference probably significantly increases $10^3\ln \beta$ of $^{26}\text{Mg}/^{24}\text{Mg}$ of the carbonate mineral with 3.12 mol% Mg relative to the one with 6.25 mol% Mg.

4.3. Implications for Mg and Ca isotopic compositions of carbonates

Stable isotopic compositions of carbonates provide important records on many low temperature geochemical processes. Their isotopic compositions are indispensable
for understanding these processes. $\delta^{26/24}$Mg values of precipitated marine Mg-calcites vary from $-5\%$ to $-1\%$ (Galy et al., 2002; de Villiers et al., 2005; Tipper et al., 2006a,b; Buhl et al., 2007; Hippler et al., 2009) and $\delta^{26/24}$Mg values of dolomites range from about $-2\%$ to $-1\%$ (Galy et al., 2002; Chang et al., 2003; Jacobson et al., 2010). These $\delta^{26/24}$Mg variations of Mg-calcite (~4$\%$) and dolomite (~1$\%$) are ascribed to kinetic effects (Immenhauser et al., 2010) or unconfirmed “vital effects” (Chang et al., 2004). According to our calculated results, although kinetic effects are potentially important, the concentration effect on Mg and Ca isotope fractionation should be also considered. The variation of $10^3 \ln z_{\text{minerals-dolomite}}$ of Mg isotopes between Mg-calcite and dolomite caused by the difference

### Table 2

Polynomial fitting parameters of the calculated reduced partition function ratios ($10^3 \ln \beta$) of $^{26}$Mg/$^{24}$Mg and $^{44}$Ca/$^{40}$Ca for calcite-type carbonate minerals with variable Mg/(Mg + Ca) and aragonite.

<table>
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<th>Element</th>
<th>Minerals</th>
<th>Mg/(Mg + Ca)</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
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<td></td>
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<td>2.34829</td>
<td>-0.01509</td>
<td>2.07E-04</td>
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$10^3 \ln \beta = ax + bx^2 + cx^3$, where $x = 10^6/T^2$. $T$ is temperature in Kelvin. All polynomial fittings are performed between 273 K to 2500 K.

![Fig. 6. Temperature dependence of the $10^3 \ln z_{\text{minerals-dolomite}}$ of (a) Mg isotopes and (b) Ca isotopes. Calcite-type carbonate minerals are always enriched in lighter Mg isotopes and heavier Ca isotopes relative to dolomite. Aragonite is enriched in light Ca isotopes relative to calcite-type carbonate minerals.](image-url)
of Mg content could be up to 4‰, and a small change of Mg/(Mg + Ca) in dolomite from 0.5 to 0.6 could account for 1‰ variation in $10^3 \ln a$ at 300 K (Fig. 7). Furthermore, the concentration effect may play an important role in controlling Mg isotope fractionation during carbonates precipitation. Li et al. (2012) measured the Mg isotope fractionation between aqueous solution and Mg-bearing calcite with Mg/(Mg + Ca) ranging from 0.008 to 0.149. According to our calculated results, $10^3 \ln a$ of $^{26}\text{Mg}/^{24}\text{Mg}$ between Mg$_{1/12}$Ca$_{11/12}$CO$_3$ and Mg$_{1/6}$Ca$_{5/6}$CO$_3$ is up to $2.1$‰. Therefore, beyond the possible kinetic effect during the experiments, further research on concentration effect is also needed to understand this discrepancy.

Ca isotopic compositions of limestone and dolomite were reported in previous studies (Kasemann et al., 2005; Steuber and Buhl, 2006; Jacobson and Holmden, 2008; Holmden, 2009). A large offset of $\delta^{44/40}\text{Ca}$ between dolomite ($1.66‰$) and limestone ($-1.05‰$) was observed, which shows a negative linear correlation with Mg/(Mg + Ca) ranging from 0 to 0.474 (Holmden, 2009). Based on our calculated results in Fig. 7b, the observed $\delta^{44/40}\text{Ca}$ difference (0.6‰) between limestone and dolomite may be related to the concentration effect on inter-mineral isotopic fractionation between Mg$_{1/12}$Ca$_{11/12}$CO$_3$ and dolomite at low temperature (Fig. 7b).

Finally, Ca and Mg isotopic compositions in marine carbonates have been used to constrain geochemical processes related to paleo-oceanography. For example, $\delta^{44/40}\text{Ca}$ variations of marine carbonates are used to infer the changes of global Ca cycling (De La Rocha, 2000; Heuser et al., 2005), and a simply steady-state model of Mg isotopes was used to constrain the oceanic cycling of Mg (Tipper et al., 2006b; Li et al., 2015). Because of the large concentration effects on equilibrium Mg and Ca isotopic fractionation among carbonate minerals, cautions need to be exercised in future studies.

### Table 3

Polynomial fitting parameters of $10^3 \ln a_{\text{minerals-dolomite}}$ of $^{26}\text{Mg}/^{24}\text{Mg}$ and $^{44}\text{Ca}/^{40}\text{Ca}$. Minerals include calcite-type carbonate minerals with variable Mg/(Mg + Ca) and aragonite.

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$10^3 \ln a_{\text{minerals-dolomite}} = ax^2 + bx^3$, where $x = 10^6/T^2$. $T$ is temperature in Kelvin. All polynomial fittings are performed between 273 K to 2500 K.
5. CONCLUSIONS

Our DFT calculations reveal that Ca–O and Mg–O bond lengths in carbonates are sensitive to their Mg and Ca contents. Mg–O bond length decreases rapidly with increasing Mg/(Mg + Ca) when Mg/(Mg + Ca) < 1/2. However, when Mg/(Mg + Ca) > 1/2, Mg–O bond length increases with increasing Mg/(Mg + Ca), because Mg atoms begin to occupy the Ca sites of dolomite, which results in much longer Mg–O bond length than Mg atoms in Mg sites of dolomite. Ca–O bond length increases slightly with increasing Mg/(Mg + Ca) when Mg/(Mg + Ca) < 1/2, then it decreases with increasing Mg/(Mg + Ca) when Mg/(Mg + Ca) > 1/2, with dolomite having the longest Ca–O bond and the shortest Mg–O bond lengths among carbonate minerals. Such concentration effect on Ca–O bond length is most prominent when Ca/(Mg + Ca) < 1/6.

The calculated equilibrium fractionation factors of Mg-Ca isotopes among carbonate minerals (MgCacalcite with various Mg/(Mg + Ca) and dolomite can be up to 4‰, comparable to the observed range of δ26/24Mg in marine Mg-calcite. A small change of Mg/(Mg + Ca) in dolomite from 0.5 to 0.6 could produce 1‰ variation in δ26/24Mg at 300 K. Therefore, the concentration effect in carbonate minerals should be taken into account when applying the isotope fractionation coefficients to understand geochemical processes.

ACKNOWLEDGMENTS

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APPENDIX A. SUPPLEMENTARY MATERIAL

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.gca.2017.03.023.

REFERENCES


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