**Quiz 1:** Assume $\alpha$ and $\gamma$ are limit ordinals such that $\exists \beta : \gamma \xrightarrow{\text{unbounded \ nondecreasing}} \alpha$. Prove that $cf(\alpha) \leq cf(\gamma)$.

Feel free to cite HW#3 that a suitable composition of unbounded, nondecreasing maps is unbounded. However, otherwise complete the proof; in particular, prove that the skipping function actually works (has domain equal to some ordinal $\leq cf(\gamma)$, etc.) Set up and end the proof appropriately.

**Quiz 2:** Assume $\alpha$ and $\gamma$ are limit ordinals such that $\exists \beta : \gamma \xrightarrow{\text{unbounded \ nondecreasing}} \alpha$. Prove that $cf(\gamma) \leq cf(\alpha)$.

**Quiz 3:** Prove that if $\kappa$ is an infinite cardinal, then $\kappa < \kappa^{cf(\kappa)}$.

$\kappa^{cf(\kappa)}$ involves cardinal exponentiation and is therefore a cardinal. It is the cardinal $|\kappa|^{cf(\kappa)}$ where $cf(\kappa)$ is the set of all functions from $cf(\kappa)$ into $\kappa$.

**Quiz 4:** Assume that $\kappa$ is an infinite cardinal such that there exist $\lambda < \kappa$ and $\langle S(\xi) \mid \xi < \lambda \rangle$ such that $|S(\xi)| < \kappa$ and $\kappa = \bigcup_{\xi < \lambda} S(\xi)$. Prove that $\kappa$ is singular, i.e. show that $cf(\kappa) < \kappa$.

**Note:** There is no Quiz 5.

**Quiz 6:** Prove $\aleph_\alpha \cdot \aleph_\alpha = \aleph_\alpha$ for any ordinal $\alpha$. Feel free to assume the base step for $\alpha = 0$ (by instructor).

**Quiz 7:** Let $\kappa$ be a regular, uncountable cardinal and $\langle C_i \mid i \in I \rangle$ be a sequence of closed subsets of $\kappa$. Show $\bigcap_{i \in I} C_i$ is also closed.

**Quiz 8:** Let $\kappa$ be a regular uncountable cardinal.

a) Prove that the intersection of two club subsets of $\kappa$ is also club.

b) Give an example that verifies the intersection of two unbounded subsets of $\kappa$ need not be unbounded.

**Quiz 9:** Let $\kappa$ be a regular uncountable cardinal.

a) Prove that if $\lambda < \kappa$ and $\langle C_\alpha \mid \alpha < \lambda \rangle$ is a sequence of club subsets of $\kappa$, then $\bigcap_{\alpha < \lambda} C_\alpha$ is club.

b) Give an example that verifies that part (a) is false if "$\lambda < \kappa$" is replaced by $\lambda = \kappa$.

**Quiz 10:** Let $\kappa$ be a regular, uncountable cardinal and $\langle C_\alpha \mid \alpha < \kappa \rangle$ be a sequence of club subsets of $\kappa$. Show $\Delta_{\alpha < \kappa} C_\alpha$ is also club.

**Quiz 11:** Prove Fodor's Theorem: Let $\kappa$ be a regular uncountable cardinal. If $f$ is a regressive function on a stationary set $S \subseteq \kappa$, then there exists a stationary $T \subseteq S$ such that $f$ restricted to $T$ is constant.

**Quiz 12:** Recall the following theorem: Let $\kappa$ be a regular, uncountable cardinal. Every stationary subset of $E^\kappa_\omega$ is the union of $\kappa$ pairwise disjoint stationary sets. State and prove the Main Lemma to this theorem.

**Quiz 13:** Let $\kappa$ be a regular, uncountable cardinal and let $W$ be a stationary subset of $E^\kappa_\omega$. For each $\alpha \in W$, let $f_\alpha : \omega \xrightarrow{\text{increasing \ unbounded}} \alpha$. Assume the Main Lemma: there exists $i < \omega$ such that for all $\eta < \kappa$, we have $W_{\geq \eta} = \{ \alpha \in W \mid f_\alpha(i) \geq \eta \}$ is stationary. Using the Main Lemma, prove that $W$ is the union of $\kappa$ pairwise disjoint stationary sets.
Quiz 14: Let $\kappa$ be a regular, uncountable cardinal, let $A$ be a stationary subset of $\kappa$, and let $A^L = \{ \alpha \in A \mid \alpha \text{ is a (infinite) limit ordinal} \}$. Recall that it is easy to show that one of two sets $A^S = \{ \alpha \in A^L \mid \text{cf}(\alpha) < \alpha \}$ and $A^R = \{ \alpha \in A^L \mid \alpha \text{ is a regular uncountable cardinal} \}$ is stationary. Assume $A^R$ is stationary, in which case one can show $W = \{ \alpha \in A^R \mid \alpha \cap A^R \text{ is not a stationary subset of } \alpha \}$ is a stationary subset of $\kappa$ so that $\forall \alpha \in W \exists F_\alpha$ ($F_\alpha$ is club in $\alpha$ and disjoint from $\alpha \cap A^R$). For each $\alpha \in W$, let $f_\alpha : \alpha \to F_\alpha$ enumerate $F_\alpha$. Prove that following Main Lemma: there exists $\xi < \kappa$ such that for all $\eta < \kappa$, we have $W^{\xi}_{\geq \eta} = \{ \alpha \in W \mid \alpha > \xi \text{ and } f_\alpha(\xi) \geq \eta \}$ is stationary.