Make sure to set up, appropriately finish each problem, cite any other results being used, cite at which points you are using the hypothesis (e.g. see problem #4), etc.

1. Prove that if $1 \leq p < \infty$ and $f, g \in L^p$, then $f + g \in L^p$, and $\|f + g\|_p \leq \|f\|_p + \|g\|_p$.

2. Prove that if $1 < p < \infty$, $1 < q < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$, $f \in L^p$ and $g \in L^q$, then $f \cdot g \in L^1$, and 
   $$\int |f \cdot g| \leq \|f\|_p \|g\|_q.$$ 

3. For $1 \leq p < \infty$, prove that the $L^p$ spaces are complete.

4. Prove that if $1 \leq p < \infty$ and $F$ is a bounded linear functional on $L^p$, then $\exists g \in L^q$ such that 
   $$F(f) = \int fg \quad \text{and} \quad \|F\| = \|g\|_q.$$ 
   As with all exam questions, make sure to set up, appropriately finish, cite at which points you use the hypothesis (in particular, for #4 make sure to cite at which points you use that $F$ is bounded), etc.