As usual, make sure to appropriately set up and end your proofs, make sure to cite any results or problems used at the appropriate places, etc. Included is a list of results 1-14 from Chapter 5 to make it easy to cite these appropriately. (Textbook: Real Analysis by H. L. Royden, Third Edition)

1. (Lemma 1) Let $E$ be a set contained in an open set $O$ of finite measure and let $J$ be a Vitali covering of $E$ which contains only closed sets that are subsets of $O$. Prove that for every $\varepsilon > 0$ there are finitely many pairwise disjoint sets $I_1, I_2, I_3, \ldots, I_N \in J$ such that $m^*(E \backslash \bigcup_{i \in N} I_i) < \varepsilon$.

2. (Theorem 3) Assume that $f$ is an increasing real-valued function on $[a, b]$. Let $E_{u,v} = \{ x : D^+ f(x) > u > v > D_- f(x) \}$. Prove $mE_{u,v} = 0$.

3. (Theorem 10) Assume that $f$ is integrable on $[a, b]$ and that $F(x) = F(a) + \int_a^x f(t) \, dt$. Prove that $F'(x) = f(x)$ for almost all $x \in [a, b]$.

4. (Lemma 13) Prove that if $f$ is absolutely continuous function on $[a, b]$ and $f'(x) = 0$ a.e., then $f$ is constant.

5. (Theorem 14) Prove that every absolutely continuous function $F$ on $[a, b]$ is an indefinite integral of its derivative, i.e. $F(x) = F(a) + \int_a^x F'(t) \, dt$. 
1. **Lemma (Vitali)** (p 98): Let $E$ be a set of finite outer measure and $J$ a collection of intervals that cover $E$ in the sense of Vitali. Then, given $\varepsilon > 0$, there is a finite disjoint collection $\{I_1, \ldots, I_N\}$ of intervals in $J$ such that $m^*\left[ E \sim \bigcup_{n=1}^{N} I_n \right] < \varepsilon$.

2. **Proposition** (p 99): If $f$ is continuous on $[a,b]$ and one of its derivatives (say $D^+$) is everywhere nonnegative on $(a,b)$, then $f$ is nondecreasing on $[a,b]$; i.e., $f(x) \leq f(y)$ for $x \leq y$.

3. **Theorem** (p 100): Let $f$ be an increasing real-valued function on the interval $[a,b]$. Then $f$ is differentiable almost everywhere. The derivative $f'$ is measurable, and $\int_a^b f'(x) \, dx \leq f(b) - f(a)$.

4. **Lemma** (p 103): If $f$ is of bounded variation on $[a,b]$, then $T^b_a = P^b_a + N^b_a$ and $f(b) - f(a) = P^b_a - N^b_a$.

5. **Theorem** (p 103): A function $f$ is of bounded variation on $[a,b]$ if and only if $f$ is the difference of two monotone real-valued functions on $[a,b]$.

6. **Corollary** (p 104): If $f$ is of bounded variation on $[a,b]$, then $f'(x)$ exists for almost all $x$ in $[a,b]$.

7. **Lemma** (p 105): If $f$ is integrable on $[a,b]$, then the function $F$ defined by $F(x) = \int_a^x f(t) \, dt$ is a continuous function of bounded variation on $[a,b]$.

8. **Lemma** (p 105): If $f$ is integrable on $[a,b]$ and $\int_a^x f(t) \, dt = 0$ for all $x$ in $[a,b]$, then $f(t) = 0$ a.e. in $[a,b]$.

9. **Lemma** (p 106): If $f$ is bounded and measurable on $[a,b]$ and $F(x) = \int_a^x f(t) \, dt + F(a)$, then $F'(x) = f(x)$ for almost all $x$ in $[a,b]$.

10. **Theorem** (p 107): Let $f$ be integrable function on $[a,b]$, and suppose that $F(x) = F(a) + \int_a^x f(t) \, dt$. Then $F'(x) = f(x)$ for almost all $x$ in $[a,b]$.

11. **Lemma** (p 108): If $f$ is absolutely continuous on $[a,b]$, then it is of bounded variation on $[a,b]$.

12. **Corollary** (p 109): If $f$ is absolutely continuous, then $f$ has a derivative almost everywhere.

13. **Lemma** (p 109): If $f$ is absolutely continuous on $[a,b]$ and $f'(x) = 0$ a.e., then $f$ is constant.

14. **Theorem** (p 110): A function $F$ is an indefinite integral if and only if it is absolutely continuous.

15. **Proposition (from chapter 4.14)** (p 88): Let $f$ be a nonnegative function which is integrable over a set $E$. Then given $\varepsilon > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with $mA < \delta$ we have $\int_A f < \varepsilon$. 

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