2. Suppose \( E \subseteq \mathbb{R} \) is infinite and there exists \( g : \omega \rightarrow^\text{onto} E \). Provide “elementary” maps that verify the set \( \text{Alg}(E) \) of algebraic numbers over \( E \) is denumerable (again without using Bernstein Schroeder). Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies that \( \text{Alg} \) is denumerable.

Indicate which maps are 1-1 and/or onto, and indicate what bijection verifies the actual equality listed.

4. Prove that for every (nonempty) perfect set \( P \subseteq \mathbb{R} \) there exists an injection \( \Psi : \{0,1\}^\omega \rightarrow P \).

5. (a) Outline the proof of the Bernstein Schroeder Theorem: If \( |A| \leq |B| \) and \( |B| \leq |A| \), then \( |A| = |B| \).

(b) If \( f : A \rightarrow^\text{1-1} B \) and \( g : B \rightarrow^\text{1-1} A \), then \( B \setminus \text{Im } f \subseteq \) and \( A \setminus \text{Im } g \subseteq \).

6. Let \( F \subseteq \mathbb{R} \) be an uncountable closed set and assume \( P \subseteq F \) is perfect (and nonempty). Use the Bernstein Schroeder Theorem and problems from Test 1 to show that \( |F| = |\mathbb{R}| = |P| = |\phi(\omega)| = |\{0,1\}^\omega| \).

13. Prove the Perfect Set Theorem: for any uncountable closed set \( F \subseteq \mathbb{R} \) there exists a perfect set \( P \) and a countable set \( C \) such that \( F = P \cup C \).

16. Assume \( \alpha > 0 \) is a limit ordinal and \( \beta : \gamma \rightarrow^\text{unbounded, nondecreasing} \alpha \). Define the strictly increasing map that “shows/witnesses” \( \text{cf}(\gamma) \leq \text{cf}(\alpha) \).