MIXED FINITE ELEMENTS FOR PARABOLIC INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract In this paper, we study mixed finite elements for parabolic integro-differential equations, and introduce a kind of nonclassical mixed projection, its optimal $L^2$ and $h^{-1}$ estimates are obtained. We define semi-discrete and full-discrete mixed finite elements for the equations, and obtain the optimal $L^2$ error estimates.

Key words Integro-differential Equations, Mixed Finite Element, Error Estimates

1 Introduction

We consider the following nonlinear parabolic integro-differential equations:

(a) $u_t = \nabla \cdot \{a(u)(\nabla u + \int_0^t \nabla u(x, \tau) d\tau)\} + fu, (x, t) \in \Omega \times (0, T],$

(b) $u(x, 0) = u_0(x), x \in \Omega,$

(c) $u(x, t) = 0, (x, t) \in \partial \Omega \times (0, T].$

where $\Omega \subset \mathbb{R}^2$ is a bounded domain with smooth boundary $\partial \Omega, T > 0, a, u_0$ and $f$ are known functions.

Problem (1.1) can arise from many physical processes such as some gas diffusion problems, heat transfer problems with memory, and etc. (see [1]).

For approximating the solution $u$, the classical finite element methods have been considered by several authors [2],[3],[4],[5] in recent years, and the author [6],[7],[8],[9] has studied classical and nonclassical finite element methods for integro-differential equations of evolution. But the mixed finite elements for problem (1.1) have not been considered by any authors. In this paper, we introduce a nonclassical mixed projection: Volterra-type mixed projection from which optimal $L^2$ error estimates can be derived for semi-discrete and full-discrete mixed finite elements for problem (1.1).

This paper is organized in the following way. In §2 we give mixed finite element formulations and some necessary preparations. The Volterra-type mixed projection will be introduced and studied in §3. Section 4 and 5 contain the error estimates for the semi-discrete and full-discrete mixed finite element approximations, respectively for problem (1.1).

Received Mar.5, 1995; revised Jan.2, 1996.