1. Let $E$ and $F$ be two events of a sample space, $S$, with $P(F) > 0$. The event $E$, is independent of the event $F$, if and only if $P(E|F) = P(E)$

2. If two events have positive probabilities ($P(E) > 0$ and $P(F) > 0$), and if the event $E$ is independent of $F$, then $F$ is also independent of $E$. So, we say that $E$ and $F$ are independent events.

3. Criterion for independent events:
   (a) $P(E \cap F) = P(E) \cdot P(F)$

4. Independence for more than two events, simply apply the criterion over and over which gives:

   $P(E_1 \cap E_2 \cap E_3 \cap \ldots \cap E_n) = P(E_1) \cdot P(E_2) \cdot P(E_3) \ldots \cdot P(E_n)$

5. Example: Given $P(E) = 0.3, P(F) = 0.2, P(E \cup F) = 0.4$. Are $E$ and $F$ independent?
   Solution: First find $P(E \cap F)$ by using $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ to get $P(E \cap F) = 0.1$
   Now apply number (3) to see if
   $P(E \cap F) \overset{?}{=} P(E) \cdot P(F)$
   $0.1 \overset{?}{=} 0.3 \cdot 0.2$
   $0.1 \neq 0.06$
   Since, $P(E \cap F) \neq P(E) \cdot P(F)$, then $E$ and $F$ are not independent.