A. Motivation

1. Suppose that someone rolls a single die, out of your sight, and tells you it came up an even number. You are then asked “What is the probability that a 2 has been rolled?” Then answer that you give is certainly affected by the information that is know to you, namely, that the die is know to have come up 2, 4, or 6. This, in effect, reduces the sample space to \( S = \{2, 4, 6\} \) from which the answer is obviously \( \frac{1}{3} \). The mathematical notation for this question is:

\[
P(\text{a two comes up} | \text{an even number has been rolled}) = \frac{1}{3},
\]

where the vertical bar is read as “given that”

B. Formulas

1. Conditional Probability is: Let \( E \) and \( F \) be events of a sample space \( S \) and suppose \( P(F) > 0 \). The conditional probability of the event \( E \), given the event \( F \), denoted by \( P(E|F) \), is defined as

\[
P(E|F) = \frac{P(E \cap F)}{P(F)}
\]

2. Product Rule: For two events \( E \) and \( F \), the probability of the event \( E \) and \( F \), namely \( P(E \cap F) \) is given by

\[
P(E \cap F) = P(F) \cdot P(E|F)
\]

C. Examples

1. If two cards are randomly drawn, in succession, without replacement, from a deck of 52, what is the probability that the second card is a spade given that the first card is a spade? In mathematical language we have: \( P(\text{second card is a spade} | \text{first was a spade}) \). Because a spade is gone, the new sample space is 51 cards including 12 spades. The answer is then \( \frac{12}{51} \).
2. One card is drawn from a deck of 52 cards. What is the probability that the card is:

(a) A king (K) given that the card drawn is a heart? \( \frac{1}{13} \)

(b) A diamond given that the card drawn was a heart or diamond? \( \frac{13}{26} = \frac{1}{2} \)

(c) A heart, given that the card drawn was a diamond or a 10? \( P(H|10 \cup D) = \frac{1}{16} \)

3. The senior class of Podunko High has 80 members, 50 of whom are taking English \((E)\), 30 math \((M)\) and 20 both subjects. What is the probability that a senior taking English, given that the senior is taking math?

\[
P(E|M) = \frac{P(E \cap M)}{P(M)} = \frac{\frac{20}{80}}{\frac{30}{80}} = \frac{20}{30} = \frac{2}{3}
\]

4. A tree diagram is often helpful in working conditional probability problems. You would multiply the branch probabilities.