A. Linear Programming Problems

1. Def’n: (Linear Programming Problem)
   A linear programming problem (lpp) in two variables, \(x\) and \(y\), consists of maximizing or minimizing an objective function
   \[z = Ax + By\]
   where \(A, B \in \mathbb{R}\), not both zero, subject to constraints expressible as a system of linear inequalities.

2. Def’n: To maximize(minimize) \(z = Ax + By\) means to locate the points that result in the largest(or smallest) value of \(z\). Only the points that obey all the constraints are potential solutions. These points are called feasible points.

3. A solution to a lpp is a feasible points, together with the value of the objective function at that point, which maximizes(or minimizes) the objective function. If none of the feasible points maximizes(or minimizes) the obj func, or there are no feasible points, then the lpp has no solution.

B. Solving a lpp

1. Theorem(Criteria for the Existence of a Solution)
   Consider a lpp with the set \(R\) of feasible points and objective function \(z = Ax + By\).
   (a) If \(R\) is bounded, then \(z\) has both a max and min value on \(R\)
   (b) If \(R\) is unbounded, \(A, B > 0\), and the constraints include \(x, y \geq 0\), then \(z\) has a min on \(R\) but not a max.
   (c) If \(R = \emptyset\), then the lpp has no solution and \(z\) has neither a max nor a min.

2. Theorem(Fundamental Theorem for LPP with Two Variables)
   Consider a lpp with the set \(R\) of feasible points and objective function \(z = Ax + By\), where \(x, y \geq 0\).
   If a lpp has a solution, it is located at a corner point of the set \(R\) of feasible points; if a lpp has multiple solutions, at least one of them is located at a corner point of the set \(R\) of feasible points. In either case the corresponding value of the objective function is unique.

3. Steps for Solving a LPP
   If a lpp has a solution, follow these steps to find it:
   S1: Write an expression for the quantity to be maximized or minimized. (the objective function)
   S2: Determine all the constraints and graph the set of feasible points.
   S3: List the corner points of the feasible points.
   S4: Determine the value of the objective function at each corner point.
   S5: Select the maximum or minimum value of the objective function.