I have taken the time prior to the semester to compile a collection of typed-up notes. Granted these notes are not perfect, nor necessarily complete. I have primarily taken the necessary information from each section and collected it into one place. For the most part I have excluded examples, unless I felt that they were necessary to show particular concepts. I feel that there are plenty of examples in the text for the students to look at on their own, and the examples I do in class are from the problems in the textbook.

Note to the Student. These notes are not a substitute to reading the textbook. They are only to be used as a guideline for the instructor. I have left out the examples and other material, which can only be gained by looking at the sections. It will be helpful to you, if you read the sections that are to be covered, prior to the class meeting.

These note are by no means perfect. I am constantly looking for ways to improve them. If you notice anything that is incorrect or needs to be changed or added, please let me know. My email address is fisher18@unlv.nevada.edu.
1. Types of Numbers

   (a) Natural Numbers (Counting Numbers): \( N = \{1, 2, 3, 4, 5, 6, \ldots \} \)

   (b) Whole Numbers: \( W = \{0, 1, 2, 3, 4, 5, 6, \ldots \} \)

   (c) Integers: \( Z = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots \} \). You can use a number line to express the integers.

   (d) Rational Numbers: A rational number is any number that can be written in the form \( \frac{a}{b} \) where \( a, b \in Z \) and \( b \neq 0 \). (The letter Q represents the set of all rational numbers)

   (e) Irrational Numbers: An irrational number is any number that can be written as an infinite, non-repeating decimal.

   (f) Real Numbers: The rational and irrational numbers

2. Real Number Lines

   (a) There is a one-to-one correspondence between the real numbers and the points on a line

3. Sets and Set-Builder Notation

   (a) Definitions

      i. Set: any collection of objects

      ii. Element: Member of a set.

      iii. Roster Method: listing of the objects in the set

      iv. Element of Notation: We use the symbol \( \in \) which is read “is an element of” and is used to indicate that a particular number belongs to a set.

      v. Set-Builder Notation: Describing properties held by all members and no others. The notation used is \( \{x | \} \) where | is read “such that.”

      vi. Finite: If the elements in a set can be counted, then the set is finite, otherwise the set is infinite.

      vii. Empty (null) set: Set with no elements. Notation: \( \emptyset \) or \( \{\} \)

   (b) Inequality Symbols

      \(< \) less than \( \leq \) less than or equal to

      \(> \) greater than \( \geq \) greater than or equal to

   (c) Union and Intersection

      i. Union \( (A \cup B) \): set of all elements in \( A \) OR \( B \). Example: \( \{x|x < a \text{ or } x > b\} \) can be written as \( \{x|x < a\} \cup \{x|x > b\} \)

      ii. Intersection \( (A \cap B) \): set of all elements in both \( A \) AND \( B \). Example: \( \{x|x a a \text{ and } x < b\} \) can be written as \( \{x|x > a\} \cap \{x|x < b\} \)
(d) Field Properties of the Real Numbers

<table>
<thead>
<tr>
<th>Addition</th>
<th>Name of property</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1: $a + b$ is a real number</td>
<td>Closure</td>
<td>M1: $a \cdot b$ is a real number</td>
</tr>
<tr>
<td>A2: $a + b = b + a$</td>
<td>Commutative</td>
<td>M2: $a \cdot b = b \cdot a$</td>
</tr>
<tr>
<td>A3: $(a + b) + c = a + (b + c)$</td>
<td>Associative</td>
<td>M3: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$</td>
</tr>
<tr>
<td>A4: $a + 0 = 0 + a = a$</td>
<td>Identity</td>
<td>M4: $a \cdot 1 = 1 \cdot a = a$</td>
</tr>
<tr>
<td>A5: $a + (-a) = 0$</td>
<td>Inverse</td>
<td>M5: $a \cdot \frac{1}{a} = 1$</td>
</tr>
</tbody>
</table>

D Distributive Property: $a \cdot (b + c) = a \cdot b + a \cdot c$ or $(b + c) \cdot a = b \cdot a + c \cdot a$

(e) Properties of Inequality (Order) For real numbers $a, b$ and $c$:

O1 Trichotomy Property: Exactly one of the following is true: $a < b$, $a = b$, $a > b$.

O2 Transitive Property: If $a < b$ and $b < c$, then $a < c$. 

version 0.1
1. Absolute Value

(a) Def’n: For any real number \( x \), we have:
\[
|x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases}
\]

2. Addition and Subtraction

(a) Rules for Adding Real Numbers

i. To add two real numbers with like signs, add their absolute values and use the common sign.

ii. To add two real numbers with unlike signs, subtract their absolute values (the smaller from the larger), and use the sign of the number with the larger absolute value.

(b) Subtraction: For real numbers \( a \) and \( b \),
\[
a - b = a + (-b)
\]

3. Multiplication and Division

(a) Symbols for Multiplication

\( \cdot \) raised dot

( ) numbers inside or next to parentheses

\( \times \) cross sign

number written next to a variable

variable written next to variable

(b) Theorem: For positive real numbers \( a \) and \( b \),
\[
a(-b) = (-a)b = -ab
\]

(c) Theorem: For positive real numbers \( a \) and \( b \),
\[
(-a)(-b) = ab
\]

(d) Division: For real numbers \( a \) and \( b \) \((b \neq 0)\),
\[
\frac{a}{b} = x \text{ if and only if } a = b \cdot x
\]

(e) Rules for Multiplication and Division with Real Numbers:

i. If two nonzero numbers have like signs, then both their product and their quotient will be positive.

ii. If two nonzero numbers have unlike signs, then both their product and quotient will be negative.
(f) Multiplication Property of Zero: For any real number $a$

\[ a \cdot 0 = 0 \]

(g) Division by 0: Division by 0 is undefined.

(h) Exponent: In general for any whole number $n$ and any real number $a$,

\[ \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}} = a^n \]

where $n$ is the exponent and $a$ is the base.

(i) Rules for Order of Operations:

i. Simplify within symbols of inclusion (parentheses, brackets, braces, fraction bar, absolute value bars) beginning with the innermost symbols.

ii. Find any powers indicated by exponents or roots

iii. Multiply or divide from left to right.

iv. Add or subtract from left to right.
1. Combining Like Terms

(a) Def’n: An algebraic expression is an expression involving variables and numbers using any of the operations of addition, subtraction, mult, div as well as exponents and roots.

(b) Def’n: A term is an expression that involves only multiplication and/or division with constants and variables.

(c) Def’n: A constant term is a term that consists of only a real number.

(d) Def’n: Like terms (or similar terms) are those terms that have the same variable factors with the same exponents.

(e) Def’n: The numerical factor of a term is called the coefficient.

(f) In order to combine like terms, we use the distributive property.

2. First-Degree (or Linear) Equations

(a) First-Degree Equation: An equation of the form $ax + b = c$, where $a$, $b$ and $c$ are real numbers and $a \neq 0$, is called a first-degree (or linear) equation in $x$.

(b) Theorem: Every first-degree equation has exactly one solution.

(c) Addition Property of Equality: If the same algebraic expression is added to both sides of an equation, the new equation has the same solution as the original equation. Symbolically, if $A$, $B$, and $C$ are algebraic expressions, and if $A = B$ then $A + C = B + C$ and $A - C = B - C$.

Note: Subtracting $C$ is the same as adding its opposite $-C$.

(d) Multiplication Property of Equality: If both sides of an equation are multiplied by the same nonzero algebraic expression, the new equation has the same solution as the original equation. Symbolically, if $A$, $B$, and $C$ are algebraic expressions, and if $A = B$ then $AC = BC$ and $A/C = B/C$.

Note: Dividing by $C$ is the same as multiplying by its reciprocal $1/C$.

(e) Def’n: Two equations that have the same solution set are equivalent.

(f) To Solve a First-Degree (or Linear) Equation in One Variable:

i. Simplify each side of the equation by removing any grouping symbols and combining like terms. (In some cases, you may want to multiply both sides of the equation by a constant to clear fractional or decimal coefficients.)

ii. Use the addition property of equality to add the opposites of constants or variable expressions so that variable expressions are on one side of the equation and constants on the other.
iii. Use the multiplication property of equality to multiply both sides by the reciprocal of the coefficient of the variable (that is, divide both sides by the coefficient) so that the new coefficient is 1.

iv. Check your answer by substituting it into the original equation.

3. Condition Equations, Identities and Contradictions

(a) Def’n: If an equation has a finite number of solutions, the equation is said to be a conditional equation.

(b) Def’n: Simplifying an equation to a statement that is always true, such as \(0 = 0\). In this case the original equation has an infinite number of solutions and is called an identity.

(c) Def’n: If the equation simplifies to a statement that is never true, then the original equation is called a contradiction and its solution is the \(\emptyset\).

4. Absolute Value Equations

(a) Absolute Values: For any real number \(x\),

\[
|x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases}
\]

(b) Solving Absolute Value Equations:

For \(c > 0\)

i. If \(|x| = c\), then \(x = c\) or \(x = -c\).

ii. If \(|ax + b| = c\), then \(ax + b = c\) or \(ax + b = -c\).

(c) Two Absolute Values: If \(|a| = |b|\), then either \(a = b\) or \(a = -b\). More generally, if \(|ax + b| = |cx + d|\), then either \(ax + b = cx + d\) or \(ax + b = -(cx + d)\).
1. Def’n: A formula is an equation that represents a general relationship between two or more quantities or measurements.

2. See book for formula listings

3. Do example problems
1. Strategy for Solving Word Problems:

   (a) Understand the problem.
       i. Read the problem carefully. (Read it several times if necessary).
       ii. If it helps, restate the problem in your own words.

   (b) Devise a plan.
       i. Decide what is asked for; assign a variable to the unknown quantity. Label this variable so you know exactly what it represents.
       ii. Draw a diagram or set up a chart whenever possible.
       iii. Write an equation that relates the information provided.

   (c) Carry out the plan.
       i. Study your picture or diagram for insight into the solution.
       ii. Solve the equation.

   (d) Look back over your results.
       i. Does your solution make sense in terms of the wording of the problem?
       ii. Check your solution in the equation.

2. Look for key words in the problems, such as,

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
<th>Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>add</td>
<td>subtract</td>
<td>multiply</td>
<td>divide</td>
<td>gives</td>
</tr>
<tr>
<td>sum</td>
<td>difference</td>
<td>product</td>
<td>quotient</td>
<td>represents</td>
</tr>
<tr>
<td>plus</td>
<td>minus</td>
<td>times</td>
<td>ratio</td>
<td>amounts to</td>
</tr>
<tr>
<td>more than</td>
<td>less than</td>
<td>twice</td>
<td></td>
<td>is / was</td>
</tr>
<tr>
<td>increased by</td>
<td>decreased by</td>
<td>of(with fractions and percents)</td>
<td></td>
<td>is the same as</td>
</tr>
</tbody>
</table>
1. Linear Inequalities

(a) Interval Notation:

<table>
<thead>
<tr>
<th>Name of Interval</th>
<th>Algebraic Notation</th>
<th>Interval Notation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open Interval</td>
<td>( a &lt; x &lt; b )</td>
<td>((a, b))</td>
<td></td>
</tr>
<tr>
<td>Closed Interval</td>
<td>( a \leq x \leq b )</td>
<td>([a, b])</td>
<td></td>
</tr>
<tr>
<td>Half-open</td>
<td>( a \leq x &lt; b )</td>
<td>([a, b))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( a &lt; x \leq b )</td>
<td>((a, b])</td>
<td></td>
</tr>
</tbody>
</table>

(b) Linear Inequalities: For real numbers \( a, b \) and \( c \) \( (a \neq 0) \), \( ax + b < c \) is called a linear inequality in \( x \). (The definition holds if \( \leq, >, \) and \( \geq \) is used instead of \(<\).)

(c) NOTE: Multiplying or dividing both sides of an inequality by a negative number reverses the sense of the inequality.

(d) To Solve a Linear Inequality

i. Simplify each side of the inequality by removing any grouping symbols and combining like terms.

ii. Add the opposites of constants and/or variable expressions to both sides so that variables are on one side and constants are on the other.

iii. Divide both sides by the coefficient of the variable and

A. leave the direction of the inequality if the coefficient is positive; or

B. reverse the direction of the inequality if the coefficient is negative.

(e) Addition Property of Inequalities: If the same algebraic expression is added to both sides of an inequality, the new inequality in equivalent to the original inequality.

If \( A < B \), then \( A + C < B + C \)

If \( A < B \), then \( A - C < B - C \)

(f) Multiplication Property of Inequalities: If both side of an inequality are multiplied (or divided) by the same positive expression, then the new inequality is equivalent to the original inequality.

If \( A < B \) and \( C > 0 \) then \( AC < BC \)

If \( A < B \) and \( C > 0 \), then \( \frac{A}{C} < \frac{B}{C} \)

If both sides of an inequality are multiplied (or divided) by the same negative expression and the inequality is reversed, then the new inequality is equivalent to the original inequality.

If \( A < B \) and \( C < 0 \) then \( AC > BC \)

If \( A < B \) and \( C < 0 \), then \( \frac{A}{C} > \frac{B}{C} \)

2. Inequalities with Three Parts(Examples)
3. Absolute Value Inequalities

(a) Solving Absolute Value Inequalities: For $c > 0$;
   i. If $|x| < c$, then $-c < x < c$.
   ii. If $|ax + b| < c$, then $-c < ax + b < c$.
   Note: These inequalities are true if $<$ is replaced by $\leq$.

(b) Solving Absolute Value Inequalities: For $c > 0$;
   i. If $|x| > c$, then $x > c$ or $x < -c$.
   ii. If $|ax + b| < c$, then $ax + b > c$ or $ax + b < -c$.
   Note: These inequalities are true if $>$ and $<$ are replaced by $\geq$ and $\leq$ respectively.
1. The Exponent 1
   
   (a) The Exponent 1: For any real number \( a \), \( a = a^1 \)

2. Product Rule for Exponents
   
   (a) Product Rule for Exponents: If \( a \) is a nonzero real number and \( m \) and \( n \) are integers, then
   
   \[ a^m \cdot a^n = a^{m+n} \]

3. The Exponent 0
   
   (a) The Exponent 0: If \( a \) is a nonzero real number, then
   
   \[ a^0 = 1 \]

   Note: The expression \( 0^0 \) is undefined.

4. Negative Exponents
   
   (a) Negative Exponents: If \( a \) is a nonzero real number \( n \) is an integer, then
   
   \[ a^{-n} = \frac{1}{a^n} \]

   Remember that a negative exponent indicates a fraction, not a negative number.

5. Quotient Rule for Exponents
   
   (a) Quotient Rule for Exponents: If \( a \) is a nonzero real number and \( m \) and \( n \) are integers, then
   
   \[ \frac{a^m}{a^n} = a^{m-n} \]

6. Power Rule for Exponents
   
   (a) Power Rule for Exponents: If \( a \) is a nonzero real number and \( m \) and \( n \) are integers, then
   
   \[ (a^m)^n = a^{mn} \]
1. Properties of Exponents

(a) Power Rule for Products: If $a$ and $b$ are nonzero real numbers and $n$ is an integer, then

$$(ab)^n = a^n b^n$$

(b) Power Rule for Fractions: If $a$ and $b$ are nonzero real numbers and $n$ is an integer, then

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Note: Fractions with negative exponents,

$$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$$

2. Scientific Notation

(a) Scientific Notation: If $N$ is a decimal number, then in scientific notation

$$N = a \times 10^n$$

where $1 \leq a < 10$ and $n$ is an integer
1. Cartesian Coordinate System
   (a) Def’n: An ordered pair is of the form \((x, y)\).
   (b) Origin
   (c) x-axis
   (d) y-axis
   (e) Quadrants

2. Solutions to Equations
   (a) independent variable is \(x\)
   (b) Dependent variable is \(y\).
   (c) Solution of an Equation in Two Variables: The solution (or solution set) of an equation in two variables, \(x\) and \(y\), consists of all those ordered pairs of real numbers \((x, y)\) that satisfy the equation.
   (d) Standard Form of a Linear Equation: Any equation of the form
   \[Ax + By = C\]
   where \(A\) and \(B\) are not both equal to 0
   is called the standard form of a linear equation.
   (e) To Graph a Linear Equation in Two Variables:
      i. Locate any two points that satisfy the equation.
      ii. Plot these two points on a Cartesian Coordinate system.
      iii. Draw a straight line through these two points.
   (f) Locate the \(y\)-intercept and \(x\)-intercept
      i. To find the \(x\)-intercept, set \(x = 0\) and solve for \(y\).
      ii. To find the \(y\)-intercept, set \(y = 0\) and solve for \(x\).
1. Slope of a Line
   (a) Slope of a Line: Given two points \((x_1, y_1)\) and \((x_2, y_2)\) on a line. Then the slope of the line containing these points is given by:
   \[
   m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } x_1 \neq x_2
   \]

2. Slope-Intercept Form: \(y = mx + b\)
   (a) Slope-Intercept Form: Any equation of the form
   \[
   y = mx + b
   \]
   is called the slope-intercept form of the equation of a line. The slope of the line is \(m\) and the \(y\)-intercept is the point \((0, b)\).

3. Horizontal and Vertical Lines
   (a) Horizontal and Vertical Lines:
   i. Any equation of the form \(y = b\) represents a horizontal line with slope 0.
   ii. Any equation of the form \(x = a\) represents a vertical line with undefined slope.
1. The Point-slope Form: \( y - y_1 = m(x - x_1) \)

   (a) Point-slope Form: An equation of the form

   \[
y - y_1 = m(x - x_1)
   \]

   is called the point-slope form for the equation of a line that contains the point \((x_1, y_1)\) and has slope \(m\).

2. Parallel Lines and Perpendicular Lines

   (a) Parallel Lines: Two lines (neither vertical) are parallel if and only if they have the same slope. All vertical lines are parallel.

   (b) Perpendicular Lines: Two lines (neither vertical) are perpendicular if and only if their slopes are negative reciprocals of each other

   \[
m_1 = -\frac{1}{m_2} \quad \text{or} \quad m_1m_2 = -1
   \]

   Vertical lines are perpendicular to horizontal lines.
1. Relations and Functions

(a) Relation, Domain, and Range:
   i. A relation is a set of ordered pairs of real numbers
   ii. The domain, $D$, of a relation is the set of all first coordinates in the relation.
   iii. The range, $R$, of a relation is the set of all second coordinates in the relation.

(b) Function: A function is a relation in which each domain element has a unique range element.

   OR

   A function is a relation in which each first-coordinate appears only once.

   OR

   A function is a relation in which no two ordered pairs have the same first coordinates

(c) Vertical Line Test: If any vertical line intersects the graph of a relation at more than one point, then the relation graphed is not a function.

2. Linear Functions

(a) Linear Functions: A linear function is a function represented by an equation of the form

\[ y = -\frac{A}{B}x + \frac{C}{B} \quad \text{(or } y = mx + b \text{) where } B \neq 0 \]

The domain of a linear function is the set of all real numbers.
1. Graphing Linear Inequalities

(a) Two Methods for Graphing Linear Inequalities:
   First, graph the boundary line (dashed if the inequality is \(< \) or \(> \), solid if the inequality is \(\leq \) or \(\geq \)).
   Method 1
   i. Test any one point obviously on one side of the line.
   ii. If the test-point satisfies the inequality, shad the half-plane on that side of the line. Otherwise, shade the other half-plane.

Method 2
   i. Solve the inequality for \(y\) (assuming that the line is not vertical).
   ii. If the solution shows \(y < \) or \(y \leq \), then shade the half-plane below the line.
   iii. If the solution shows \(y > \) or \(y \geq \), then shade the half-plane above the line.
   (Note: If the boundary line is vertical, use method 1)
1. Preliminary Information

(a) Two (or more) linear equations considered at one time are said to form a system of linear equations.

(b) A system of linear equations is consistent if it has exactly one solution.

(c) A system of linear equations is inconsistent if it has no solutions.

(d) A system of linear equations is dependent if it has an infinite number of solutions.

2. The Graphing Method

(a) To solve a system of linear equations in two variables by graphing:
   i. Graph both lines on the same set of axes, and
   ii. observe the point of intersection (if there is one)

(b) Note:
   i. If the slopes of the two lines are different, then the lines will intersection in one and only one point.
   ii. If the lines are distinct and have the same slope, then the lines will be parallel and have no solution.
   iii. If the lines are the same line, the the system is dependent.

(c) The Substitution Method
   i. To Solve a System of Linear Equations by Substitution
      A. Solve one of the equations for one of the variables
      B. Substitute the resulting expression into the other equation.
      C. Solve this new equation, if possible, and then substitute back into one of the original equations to find the value of the other variable

(d) The Addition Method
   i. To solve a System of Linear Equations by Addition:
      A. Write the equations one under the other so that like terms are aligned
      B. Multiply all terms of one equation by a constant (and possibly all terms of the other equation by another constant) so that two like terms have opposite coefficients.
      C. Add like terms and solve the resulting equation, if possible. Then, back substitute into one of the original equations to find the value of the other variable.
Work Examples.
1. To Solve a System of Two Linear Inequalities:

   (a) Graph the boundary lines for both half-planes
   
   (b) Shade the region that is common to both of these half-planes. (This region is called the inter-
       section of the two half-planes.)
   
   (c) To check, pick one test-point in the intersection and verify that it satisfies both inequalities.
       Note: If there is no intersection, the system is inconsistent and has no solution.
1. Monomial
   (a) Def’n: A monomial in $x$ is an expression of the form
   \[ kx^n \]
   where $n$ is a whole number and $k$ is any real number. $n$ is called the degree of the monomial, and $k$ is the coefficient.
   (b) The degree of a monomial in more than one variable is the sum of the exponents of its variable.
   (c) A nonzero constant is a monomial of degree 0
   (d) Since there is more than one way to write 0 with a variable, it is considered a monomial with no degree.

2. Polynomials
   (a) Any monomial or algebraic sum of monomials is a polynomial.
   (b) The degree of a polynomial is the largest of the degrees of its terms after like terms have been combined.
   (c) Usually written in descending order.
   (d) Classifications of Polynomials
      | Term         | Description          |
      |--------------|----------------------|
      | Monomial     | polynomial with one term |
      | Binomial     | polynomial with two terms |
      | Trinomial    | polynomial with three terms |
   (e) A polynomial can be classified in reference to its degree as follows
      If a polynomial is of
      i. degree 0 or 1, it is called a linear polynomial
      ii. degree 2, it is called a quadratic polynomial
      iii. degree 3, it is called a cubic polynomial

3. Addition of Polynomials
   (a) The sum of two or more polynomials can be found by combining like terms.

4. Subtraction of Polynomials
   (a) To find the difference of two polynomials, either
      i. add the opposite of each term being subtracted, or equivalently
      ii. use the distributive property and multiply each term being subtracted by -1, then add

5. $P(x)$ Notation and Evaluation of Polynomials
   (a) $P(x)$ is read “$P$ of $x$” and does not mean multiplication.
1. Multiplication by Monomials
   (a) In order to multiply a polynomial of two or more terms with a monomial, we will use the distributive property \( a(b + c) = ab + ac \).

2. Multiplication of Polynomials
   (a) For the product of a binomial times a binomial, we consider the distributive property in the form \((b + c)a = ba + ca\).

3. The FOIL Method
   (a) FOIL stands for first terms, outside terms, inside terms, last terms.

4. Special Products
   (a) Special Product of Polynomials
   
<table>
<thead>
<tr>
<th>Formula</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>((a + b)(a - b) = a^2 - b^2)</td>
<td>Difference of two squares</td>
</tr>
<tr>
<td>((a + b)^2 = a^2 + 2ab + b^2)</td>
<td>Perfect square trinomial</td>
</tr>
<tr>
<td>((a - b)^2 = a^2 - 2ab + b^2)</td>
<td>Perfect square trinomial</td>
</tr>
<tr>
<td>((a - b)(a^2 + ab + b^2) = a^3 - b^3)</td>
<td>Difference of two cubes</td>
</tr>
<tr>
<td>((a + b)(a^2 - ab + b^2) = a^3 + b^3)</td>
<td>Sum of two cubes</td>
</tr>
</tbody>
</table>
   
   (b) Common Error
   For products raised to a power, we have \((ab)^n = a^n b^n\). However, this rule does not apply to sums. In particular, it does not apply to binomials.

   \[
   (a + b)^2 \neq a^2 + b^2 \\
   (a - b)^2 \neq a^2 - b^2
   \]

   Remember, the squares of binomials are trinomials.
1. Division by a Monomial
   
   (a) When the denominator is a monomial, simply divide each term in the numerator by the denominator and simplify each fraction.

2. The Division Algorithm
   
   (a) The Division Algorithm: For polynomials $P$ and $D$, the division algorithm gives
   
   \[ \frac{P}{D} = Q + \frac{R}{D}, \quad D \neq 0 \]  
   
   or, in function notation,  
   
   \[ \frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}, \quad D(x) \neq 0 \]  
   
   where $Q$ and $R$ are polynomials and the degree of $R < \text{degree of } D$.

3. Synthetic Division
   
   (a) In the special case when the divisor is a first-degree polynomial of the form $(x - c)$, then we can use the process known as synthetic division.

4. The Remainder Theorem
   
   (a) The Remainder Theorem: If a polynomial, $P(x)$, is divided by $(x - c)$, then the remainder will be $P(c)$. 
1. Greatest Common Factor

   (a) To find the greatest common factor (GCF) of a polynomial:
      i. Find the expression of highest degree and largest integer coefficient that is a factor of each
         term of the polynomial
      ii. Divide this factor into each term of the polynomial resulting in another polynomial factor.
         Note: This process is called “factoring out” the common factor.

   (b) An expression is factored completely if none of its factors can be factored.

2. Basic Forms

   (a) Formulas of Squares:
      I. Difference of two squares  \( a^2 - b^2 = (a + b)(a - b) \)
      II. Perfect square trinomial  \( a^2 + 2ab + b^2 = (a + b)^2 \)
      III. Perfect square trinomial  \( a^2 - 2ab + b^2 = (a - b)^2 \)

3. \( ac \)-Method (Grouping)

   (a) Analysis of Factoring by the \( ac \)-method
      General Method
      \( ax^2 + bx + c \)
      S1: Multiply \( a \cdot c \)
      S2: Find two integers whose product is \( ac \) and whose sum is \( b \).
         If this is not possible, then the trinomial is not factorable.
      S3: Rewrite the middle term (\( bx \)) using the two number found in
         S2 as coefficients.
      S4: Factor by grouping the first two terms and last two terms.
      S5: Factor out the common binomial factor. This will give two
         binomial factors of the trinomial \( ax^2 + bx + c \)

4. FOIL Method (Trial and Error)

   (a) The FOIL method of factoring trinomials is actually applying the FOIL method of multiplication
      in reverse. We consider two basic forms:
      i. The leading coefficient is 1: \( x^2 + bx + c \)
      ii. The leading coefficient is not 1: \( ax^2 + bx + c \)

   (b) For the first form, where the leading coefficient is 1. We will be looking at the factors of \( c \) that
      add up to \( b \).

   (c) For the second form, where the leading coefficient is not 1. We will need to consider the factors
      of both \( a \) and \( c \) such that the combinations of these factors add up to \( b \).
1. Sums and Differences of Cubes
   (a) Recall from §4.2: IV. \((a - b)(a^2 + ab + b^2) = a^3 - b^3\) Difference of two cubes
       V. \((a + b)(a^2 - ab + b^2) = a^3 + b^3\) Sum of two cubes

2. Factoring by Grouping
   (a) Not all products are binomials or trinomials. For example,

   \[
   (a + b)(c + d) = (a + b)c + (a + b)d
   = ac + bc + ad + bd
   \]

   Here there are no like terms to be combined, and the product has four terms. So, in order to factor, we would need to factor by grouping.

3. Factoring with Negative Exponents: Examples
1. Solving Equations by Factoring
   (a) Zero-Product Property: If the product of two factors is 0, then one or both of the factors must be 0. That is, if \(a\) and \(b\) are real numbers,
   \[
   \text{if } a \cdot b = 0, \text{ then } a = 0 \text{ or } b = 0
   \]
   (b) Quadratic Equation: An equation that can be written in the form
   \[
   ax^2 + bx + c = 0 \quad \text{where } a, b \text{ and } c \text{ are real numbers and } a \neq 0
   \]
   is called a quadratic equation.
   (c) To Solve an Equation by Factoring
      i. Add or subtract terms so that one side of the equation is 0.
      ii. Factor the polynomial expression.
      iii. Set each factor equal to 0 and solve for the variable.

2. Finding the Equation Given the Roots
   (a) Factor Theorem: If \(x = c\) is a root of a polynomial equation in the form \(P(x) = 0\), then \(x - c\) is a factor of the polynomial \(P(x)\).

3. Consecutive Integers
   (a) Consecutive Integers: Integers are consecutive if each is 1 more than the previous integer. Three consecutive integers can be represented as \(n, n + 1,\) and \(n + 2\).
   (b) Consecutive Even Integers: Even integers are consecutive if each is 2 more than the previous even integer. Three consecutive even integers can be represented as \(n, n + 2\) and \(n + 4\).
   (c) Consecutive Odd Integers: Odd integers are consecutive if each is 2 more than the previous odd integer. Three consecutive odd integers can be represented as \(n, n + 2\) and \(n + 4\).

4. The Pythagorean Theorem
   (a) In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.
   \[
   c^2 = a^2 + b^2
   \]
1. Basic Properties

(a) Rational Expressions: A rational expression is an expression of the form \( \frac{P}{Q} \) (or in function notation, \( \frac{P(x)}{Q(x)} \)) where \( P \) and \( Q \) are polynomials and \( Q \neq 0 \).

(b) Arithmetic Rules for Rational Numbers (or Fractions)

A rational number is a number that can be written in the form \( \frac{a}{b} \) where \( a \) and \( b \) are integers and \( b \neq 0 \).

The Fundamental Principle: \( \frac{a}{b} = \frac{a \cdot k}{b \cdot k} \) where \( k \neq 0 \).

The reciprocal of \( \frac{a}{b} \) is \( \frac{b}{a} \) and \( \frac{a}{b} \cdot \frac{b}{a} = 1 \)

Multiplication: \( \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \)

Division: \( \frac{a}{b} \div \frac{c}{d} = \frac{a \cdot d}{b \cdot c} \)

Addition: \( \frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \)

Subtraction: \( \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b} \)

(c) The Fundamental Principle of Fractions: If \( \frac{P}{Q} \) is a rational expression and \( K \) is a polynomial and \( K \neq 0 \), then

\[
\frac{P}{Q} = \frac{P \cdot K}{Q \cdot K} = \frac{P \cdot K}{Q \cdot K}
\]

(d) Negative Signs

\[
-\frac{P}{Q} = \frac{-P}{Q} = -\frac{P}{Q} \quad \text{and} \quad \frac{P}{-Q} = \frac{-P}{Q} = -\frac{P}{Q} = -\frac{P}{-Q}
\]

(e) Opposites

In general,

\[
\frac{-P}{P} = -1 \quad \text{if} \quad P \neq 0
\]

In particular

\[
\frac{a - x}{x - a} = -1 \quad \text{if} \quad x \neq a
\]

(f) Common Errors (pg 334)

2. Multiplication of Rational Expressions

(a) Multiplication of Rational Expressions

\[
\frac{P}{Q} \cdot \frac{R}{S} = \frac{P \cdot R}{Q \cdot S} \quad \text{where} \quad Q, S \neq 0
\]
3. Division of Rational Expressions

(a) Multiplication of Rational Expressions

\[
\frac{P}{Q} \cdot \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} \quad \text{where } Q, R, S \neq 0
\]
1. Adding and Subtracting Rational Expressions with Common Denominators
   (a) Addition and Subtraction of Rational Expressions
   \[
   \frac{P}{Q} + \frac{R}{Q} = \frac{P + R}{Q} \quad \text{and} \quad \frac{P}{Q} - \frac{R}{Q} = \frac{P - R}{Q}
   \]
   where \( Q \neq 0 \)

2. Finding the Least Common Multiple (LCM)
   (a) To Find the LCM for a Set of Polynomials
      i. Completely factor each polynomial (including the prime factors for numerical factors)
      ii. Form the product of all distinct factors that appear, using each factor the most number of times it appears in any one factorization.

3. Adding and Subtracting Rational Expressions with Different Denominators
   (a) Procedure for Adding or Subtracting Rational Expressions with Different Denominators
      i. Find the LCD (the LCM of the denominators)
      ii. Rewrite each fraction in an equivalent form with the LCD
      iii. Add (or subtract) the numerators and keep the common denominator.
      iv. Reduce if possible.
1. Complex Fractions

(a) A complex fraction is a fraction in which the numerator and denominator are both fractions.

(b) To Simplify a Complex Fraction (First Method)
   i. Simplify the numerator and denominator separately so that the numerator and denominator are simple fractions.
   ii. Divide by multiplying by the reciprocal of the denominator.

(c) To Simplify a Complex Fraction (Second Method)
   i. Find the LCM of all the denominators in the original numerator and denominator
   ii. Multiply both the numerator and denominator by this LCM.
1. Solving Equations with Rational Expressions

(a) A rational equation is an equation that contains at least one rational expression.

(b) To Solve a Rational Equation
   
   i. Find the LCM of all the denominators of all the rational expressions in the equation.
   
   ii. Multiply both sides of the equation by this LCM. Use the distributive property if necessary.
   
   iii. Simplify both sides of the resulting equation.
   
   iv. Solve this equation.
   
   v. Check each solution in the original equation.
      (Remember that no denominator can be 0.)

2. Proportions and Similar Triangles

(a) Proportions are equations of the form \( \frac{a}{b} = \frac{c}{d} \) that state that two ratios (or rational expressions) are equal. You can solve by multiplying both sides by the LCM of the denominators.

(b) Proportions are used when working with similar geometric figures. Similar figures are figures that meet the following two conditions:
   
   i. The corresponding angles are equal
   
   ii. The corresponding sides are proportional.

(c) In similar triangles, corresponding sides are those opposite the equal angles.

(d) Notation: Given two triangles, \( \triangle ABC \sim \triangle DEF \) where \( \sim \) is read “is similar to”. The corresponding sides are proportional and

\[
\frac{AB}{DE} = \frac{BC}{EF} \quad \text{and} \quad \frac{AC}{DF} = \frac{BC}{EF} = \frac{AC}{DF}
\]

3. Solving Inequalities with Rational Expressions

(a) Procedure for Solving Inequalities with Rational Expressions
   
   i. Simplify the inequality so that one side is 0 and on the other side both the numerator and denominator are in factored form.
   
   ii. Find the points where each linear factor is zero.
   
   iii. Mark each of these points on a number line.
   
   iv. Choose a number from each indicated interval as a test value.
   
   v. The intervals where the test values satisfy the conditions of the inequality are the solution intervals.
   
   vi. Mark a solid circle for endpoints that are included and an open circle for endpoints that are not included.
1. Strategy for Solving Word Problems

(a) Read the problem carefully. Read it several times if necessary.
(b) Decide what is asked for and assign a variable to the unknown quantity.
(c) Draw a diagram or set up a chart whenever possible.
(d) Form an equation (or inequality) that relates the information provided.
(e) Solve the equation (or inequality).
(f) Check your solution with the wording of the problem to be sure it makes sense.
1. Square Roots: $\sqrt{a}$

(a) Def’n: If an integer is squared, the result is called a perfect square.

(b) Finding the square root of a given number is the reverse of squaring a number. In general, if $b^2 = a$, then $b$ is a square root of $a$.

(c) Terminology

The symbol $\sqrt{}$ is called a radical sign.

The number under the radical sign is called the radicand.

(d) Note: Every positive real number has two square roots, one positive and one negative. The positive square root is called the principal square root.

(e) Square Root: If $a$ is a nonnegative real number and $b$ is a real number such that $b^2 = a$, then $b$ is called the square root of $a$.

If $b$ is nonnegative, then we write $\sqrt{a} = b \leftarrow b$ is called the principal square root.

and $-\sqrt{a} = -b \leftarrow -b$ is called the negative square root.

(f) Note: $\sqrt{x}$ is not a real number if $x$ is negative.

2. Simplifying Expressions with Square Roots

(a) Properties of Square Roots: If $a$ and $b$ are positive real numbers, then

1. $\sqrt{ab} = \sqrt{a}\sqrt{b}$

2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(b) Simplest Form: A square root is considered to be in simplest form when the radicand has no perfect square as a factor.

3. Simplifying Square Roots with Variables

(a) Square Root of $x^2$: If $x$ is a real number, then $\sqrt{x^2} = |x|$.

Note: Is $x \geq 0$ is given, then we can write $\sqrt{x^2} = x$.

(b) Square Roots of Expressions with Even and Odd Exponents: For any real number $x$ and positive integer $m$,

$\sqrt{x^{2m}} = |x^m|$ and $\sqrt{x^{2m+1}} = x^m\sqrt{x}$

Note that the absolute value sign is necessary only if $m$ is odd. Also, note that for $\sqrt{x^{2m+1}}$ to be defined as real, $x$ cannot be negative.

Note: If $m$ is any integer, then $2m$ is even and $2m + 1$ is odd.

4. Cube Roots: $\sqrt[3]{a}$

(a) Cube Root: If $a$ and $b$ are real number such that $b^3 = a$, then $b$ is called the cube root of $a$.

We write $\sqrt[3]{a} = b \leftarrow$ the cube root.
5. Simplifying Cube Roots

(a) Simplest Form: A cube root is considered to be in simplest form when the radicand has no perfect cube as a factor.

6. Rationalizing the Denominators in Radical Expressions

(a) To Rationalize the Denominator of a Radical Expressions

i. If the denominator contains a square root, multiply both the numerator and denominator by a square root. Choose this square root so that the denominator will be a perfect square.

ii. If the denominator contains a cube root, multiply both the numerator and denominator by a cube root. Choose this cube root so that the denominator will be a perfect cube.
1. $n^{\text{th}}$ Roots: $\sqrt[n]{a} = a^{\frac{1}{n}}$

(a) For square roots if $b^2 = a$, then $b = \sqrt{a}$ (or $b = a^{\frac{1}{2}}$)
For cube roots if $b^3 = a$, then $b = \sqrt[3]{a}$ (or $b = a^{\frac{1}{3}}$)
For fourth roots if $b^4 = a$, then $b = \sqrt[4]{a}$ (or $b = a^{\frac{1}{4}}$)
For $n^{\text{th}}$ roots if $b^n = a$, then $b = \sqrt[n]{a}$ (or $b = a^{\frac{1}{n}}$)

(b) Radical Notation: If $n$ is a positive integer and $b^n = a$, then $b = \sqrt[n]{a} = a^{\frac{1}{n}}$ (assuming $\sqrt[n]{a}$ is a real number).
The expression $\sqrt[n]{a}$ is called a radical.
The symbol $\sqrt[n]{\text{ }}$ is called a radical sign.
$n$ is called the index.
a is called the radicand.
Note: If no index is given, it is understood to be 2.

(c) Special Note about the index $n$: For the expression $\sqrt[n]{a}$ (or $a^{\frac{1}{n}}$) to be a real number:
i. $n$ can be any index when $a$ is nonnegative.
ii. $n$ must be odd when $a$ is negative.
(If $a$ is negative and $n$ is even, then $\sqrt[n]{a}$ is nonreal.)

2. Rational Exponents of the form $\frac{m}{n}$: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

(a) The General Form $a^{\frac{m}{n}}$: If $n$ is a positive integer and $m$ is any integer and $a^{\frac{1}{n}}$ is a real number then
$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}}$$

In radical notation:
$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

3. Simplifying Expressions with Rational Exponents

(a) Do Examples
1. Addition and Subtraction with Radical Expressions
   (a) To add or subtract radical expression, we use the same idea as in polynomials. Where instead of adding or subtracting like terms, we are adding or subtracting like radicals.

2. Multiplication with Radical Expressions
   (a) To find the product of two binomial radical expressions, we use the FOIL method.

3. Rationalizing Denominators of Radical Expressions
   (a) Rationalizing Denominators with Sums or Differences fo Square Roots:
      i. If the denominator is of the form \( a - b \), multiply both the numerator and denominator by its conjugate \( a + b \).
      ii. If the denominator is of the form \( a + b \), multiply both the numerator and denominator by its conjugate \( a - b \).
1. Review of Solving Equations by Factoring
   (a) Zero-Factor Property: If the product of two factors is 0, then one or both of the factors must
   be zero. Symbolically, for factors \( a \) and \( b \)
   \[
   a \cdot b = 0 \quad \text{then} \quad a = 0 \text{ or } b = 0
   \]
   (b) Quadratic Equation: An equation that can be written in the form
   \[
   ax^2 + bx + c = 0 \quad \text{where } a, b \text{ and } c \text{ are real numbers and } a \neq 0
   \]
   is called a quadratic equation.
   (c) To Solve an Equation by Factoring
      i. Add or subtract terms so that one side of the equation is 0.
      ii. Factor the polynomial expression.
      iii. Set each factor equal to 0 and solve the resulting equations.
      (Note: If two of the factors are the same, then the solution is said to be a double root or a root
      of multiplicity two.)

2. Using the Definition of Square Root and the Square Root Property
   (a) Square Root Property
   If \( x^2 = c \), then \( x = \pm \sqrt{c} \).
   If \( (x - a)^2 = c \), then \( x - a = \pm \sqrt{c} \) (or \( x = a \pm \sqrt{c} \)

3. Completing the Square
   (a) In completing the square we have two things
      i. The leading coefficient (the coefficient of \( x^2 \)) is 1.
      ii. The constant term is the square of \( \frac{1}{2} \) of the coefficient of \( x \).

4. Solving Quadratic Equations by Completing the Square
   (a) To Solve a Quadratic Equation by Completing the Square
      i. If necessary, divide or multiply both sides of the equation so that the leading coefficient is 1.
      ii. If necessary, isolate the constant term on one side of the equation.
      iii. Find the constant that completes the square of the polynomial and add this constant to both sides.
      iv. Use the Square Root Property to find the solutions of the equation.

5. Writing Equations with Known Roots
   (a) Examples
1. The Quadratic Formula

(a) The quadratic formula gives the roots of any quadratic equation in terms of the coefficients $a$, $b$ and $c$ of the general quadratic equation

$$ax^2 + bx + c = 0$$

(b) Derivation:

\[
\begin{align*}
ax^2 + bx + c &= 0 \\
\frac{b}{a}x + \frac{c}{a} &= 0 \\
\frac{b}{a}x &= -\frac{c}{a} \\
\frac{b}{a}x + \frac{b^2}{4a^2} &= \frac{b^2}{4a^2} - \frac{c}{a} \\
\left(\frac{x + \frac{b}{2a}}{2a}\right)^2 &= \frac{b^2}{4a^2} - \frac{4ac}{4a^2} \\
\left(\frac{x + \frac{b}{2a}}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\
x + \frac{b}{2a} &= \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\
x + \frac{b}{2a} &= \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{align*}
\]

(c) The Quadratic Formula: For the general quadratic equation

$$ax^2 + bx + c = 0 \text{ where } a \neq 0$$

the solutions are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. The Discriminant

(a) The expression $b^2 - 4ac$ of the quadratic formula, is called the discriminant. The discriminant $b^2 - 4ac > 0$ Two real solutions

give the following information: $b^2 - 4ac = 0$ One real solution $x = \frac{-b \pm 0}{2a} = -\frac{b}{2a}$

$b^2 - 4ac < 0$ Two nonreal solutions
1. Strategy for Solving Word Problems

(a) Understand the problem
   i. Read the problem carefully. (Read it several times if necessary.)
   ii. If it helps, restate the problem in your own words.

(b) Devise a plan.
   i. Decide what is asked for; assign a variable to the unknown quantity. Label this variable so you know exactly what it represents.
   ii. Draw a diagram or set up a chart whenever possible.
   iii. Write an equation that relates the information provided.

(c) Carry out the plan.
   i. Study your picture or diagram for insight into the solution.
   ii. Solve the equation.

(d) Look back over the results
   i. Does your solution make sense in terms of the wording of the problem?
   ii. Check your solution in the equation.

2. The Pythagorean Theorem: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.
\[ c^2 = a^2 + b^2 \]

3. Projectiles: The formula \[ h = -16t^2 + v_0t + h_0 \] is used in physics and relates to the height of a projectile such as a thrown ball, a bullet, or a rocket.
   - \( h \) = height of the object, in feet
   - \( t \) = time object is in the air, in seconds.
   - \( v_0 \) = beginning velocity, in feet per second.
   - \( h_0 \) = beginning height (\( h_0 = 0 \) if the object is initially at ground level.)

4. Cost Per Person: The cost per person is found by dividing the total cost by the number of people going to the tournament.
1. Equations with Radicals

(a) Method for Solving Equations with Radicals
   S1: Isolate one of the radicals on one side of the equation. (An equation may have more than
   one radical.
   S2: Raise both sides of the equation to the power corresponding to the index of the radical.
   S3: If the equation still contains a radical, repeat S1 and S2.
   S4: Solve the equation after all of the radicals have been eliminated.
   S5: Be sure to check all possible solutions in the original equation and eliminate any extraneous
   solutions.
1. Solving Equations in Quadratic Form by Substitution:

   (a) Look at the middle term.
   (b) Substitute a first-degree variable, say, $u$, for the variable in the expression in the middle term.
   (c) Substitute the square of this variable, $u^2$, for the variable in the expression in the first term.
   (d) Solve the resulting quadratic equation for $u$.
   (e) Substitute the results back for $u$ in the beginning substitution and solve for the original variable.