Homework 19

(In Problems 17-26 let A be of dimension $3 \times 4$, B be dimension $3 \times 3$, C be of dimension $2 \times 3$, and D be of dimension $3 \times 2$. Determine which of the following expressions are defined and, for those that are, give the dimension.)

2.5.25. $DC + B$

(In Problems 27-42 use the matrices given at the top of page 97. For Problems 27-40 perform the indicated operation(s); for Problems 41 and 42 verify the indicated property.)

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 1 \\ 4 & -1 \\ 0 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 3 \\ 4 & 2 \end{bmatrix}$$

2.5.29. $BC$

2.5.39. $3CB + 4D$

2.5.41. Verify the distributive property of matrix multiplication by finding $D(CB)$ and $(DC)B$.

2.5.43. For

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 \\ 2 \\ -2 & 4 \end{bmatrix}$$

Find $AB$ and $BA$. Notice that $AB \neq BA$.

In problems 1-6 show that the given matrices are inverses of each other.

2.6.1. $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

2.6.6. $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

In problems 7-20 find the inverse of each matrix using the reduced row-echelon technique.

2.6.7. $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

In problems 21-26 show that each matrix has no inverse.
In Problems 27-34 find the inverse, if it exists, of each matrix

2.6.29. \[
\begin{bmatrix}
3 & -2 \\
0 & 4
\end{bmatrix}
\]

In Problems 39-50 solve each system of equations by the method of example 9.

2.6.39. \[
\begin{cases}
3x + 7y = 10 \\
2x + 5y = 7
\end{cases}
\]

Homework 20

In Problems 39-50 solve each system of equations by the method of example 9.

2.6.45. \[
\begin{cases}
x + y - z = 3 \\
3x - y = -4 \\
2x - 3y + 4z = 6
\end{cases}
\]

2.6.51. Show that the inverse of \[
A = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
\]
is given by the formula

\[
A^{-1} = \frac{1}{\Delta} \begin{bmatrix}
d & -b \\
-c & a \\
\Delta & \Delta
\end{bmatrix}
\]

where \(\Delta = ad - bc \neq 0\). The number \(\Delta\) is called the determinant of \(A\).

In Problems 52-55 use the result of Problem 51 to find the inverse of each matrix
2.6.53. \[
\begin{bmatrix}
1 & 5 \\
2 & 0
\end{bmatrix}
\]

(Problems 7-14 involve break-even point. In Problems 7-10 find the break-even point for the cost \( C \) of production and the revenue \( R \). Graph each result.)

1.3.7. \( C = 10x + 600 \quad R = 30x \)

1.3.8. \( C = 5x + 200 \quad R = 8x \)

1.3.9. \( C = 0.2x + 50 \quad R = 0.3x \)

1.3.10. \( C = 1800x + 3000 \quad R = 2500x \)

1.3.11. **Break-Even Point:** A manufacturer produces items at a daily cost of $0.75 per item and sells them for $1 per item. The daily operational overhead is $300. What is the break-even point? Graph your result.

**For problem 1.3.14:** Problem 13 states: Each Sunday a newspaper agency sells \( x \) copies of a certain newspaper for $2.00 per copy. The cost to the agency for each newspaper is $1.00. The agency pays a fixed cost for storage, delivery, and so on, of $200 per Sunday. How many newspapers need to be sold for the agency to break even?

1.3.14. **Profit from Selling Newspapers:** Repeat Problem 13 if the cost to the agency is $1.25 per copy and the fixed cost is $225 per Sunday.

(Problems 15-22 involve mixture problems.)

1.3.17. **Investment Problem:** Mr. Nicholson has just retired and needs $10,000 per year in supplementary income. He has $150,000 to invest in AA bonds at 10% annual interest or in Savings and Loan Certificates at 5% interest per year. How much money should be invested in each so that he realizes exactly $10,000 in extra income per year?

(In Problems 35-52 find a general equation for the line with the given properties. Write the equation in the form \( Ax + By = C \).)

1.1.35. Slope = 2; containing the point (-4, 1)

1.1.39. Containing the points (1, 3) and (-1, 2)

1.1.47. \( x \)-intercept = (2, 0); \( y \)-intercept = (0, -1)

(In Problems 53-68 find the slope and \( y \)-intercept of each line. Graph the line.)

1.1.57. \( 2x - 3y = 6 \)
(In Problems 35-44 find an equation for the line with the given properties. Express the answer using the general form or the slope-intercept form, whichever you prefer.)

1.2.35. Parallel to the line \( y = 4x \); containing the point \((-1, 2)\)

1.2.41. Perpendicular to the line \( y = 2x - 5 \); containing the point \((-1, -2)\)

1.2.47. Find the equation of the line containing the point \((-2, -5)\) and perpendicular to the line containing the points \((-2, 9)\) and \((3, -10)\).

**Homework 21**

(In Problems 11-16 each maximum problem is not in standard form. Determine if the problem can be modified so as to be in standard form. If it can, write the modified version.)

4.1.13. Maximize
\[
P = x_1 + x_2 + x_3
\]
Subject to the constraints
\[
x_1 + x_2 + x_3 \leq 6
\]
\[
4x_1 + 3x_2 \geq 12
\]
\[
x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0
\]

4.1.15. Maximize
\[
P = 2x_1 + x_2 + 3x_3
\]
Subject to the constraints
\[
-x_1 + x_2 + x_3 \geq -6
\]
\[
2x_1 - 3x_2 \geq -12
\]
\[
x_3 \leq 2
\]
\[
x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0
\]

(In Problems 17-24 each maximum problem is in standard form. For each one introduce slack variables and set up the initial simplex tableau.)

4.1.17. Maximize
\[
P = 2x_1 + x_2 + 3x_3
\]
Subject to the constraints
\[
5x_1 + 2x_2 + x_3 \leq 20
\]
\[
6x_1 + x_2 + 4x_3 \leq 24
\]
\[
x_1 + x_2 + 4x_3 \leq 16
\]
In Problems 1-8 determine which of the following statements is true about each tableau
(a) It is the final tableau
(b) It requires additional pivoting
(c) It indicates no solution to the problem.
If the answer is (a), write down the solution; if the answer is (b), indicate the pivot
element.

\[ x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0 \]

Use the simplex method to solve each maximum problem 
4.2.19. Maximize
\[ P = 2x_1 + 4x_2 + x_3 + x_4 \]
Subject to
\[2x_1 + x_2 + 2x_3 + 3x_4 \leq 12\]
\[2x_2 + x_3 + 2x_4 \leq 20\]
\[2x_1 + x_2 + 4x_3 \leq 16\]
\[x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0\]

4.2.27. **Scheduling:** Products A, B, and C are sold door-to-door. Product A costs $3 per unit, takes 10 minutes to sell (on the average), and costs $0.50 to deliver to the customer. Product B costs $5, takes 15 minutes to sell, and is left with the customer at the time of sale. Product C costs $4, takes 12 minutes to sell, and costs $1.00 to deliver. During any week a salesperson is allowed to draw up to $500 worth of A, B, and C (at cost) and is allowed delivery expenses not to exceed $75. If a salesperson’s selling time is not expected to exceed 30 hours (1800 minutes) in a week, and if the salesperson’s profit (net after all expenses) is $1 each on a unit of A or B and $2 on a unit of C, what combination of sales of A, B, and C will lead to maximum profit and what is the maximum profit?

4.2.31. **Investment:** A financial consultant has at most $90,000 to invest in stocks, corporate bonds, and municipal bonds. The average yields for stocks, corporate bonds, and municipal bonds is 10%, 8%, and 6%, respectively. Determine how much she should invest in each security to maximize the return on her investments, if she has decided that her investment in stocks should not exceed half her funds, and that twice her investment in corporate bonds should not exceed her investment in municipal bonds.

4.2.35. **Mixture Problem:** Nutt’s Nut Company has 500 pounds of peanuts, 100 pounds of pecans, and 50 pounds of cashews on hand. They package three types of 5-pound cans of nuts: can I contains 3 pounds peanuts, 1 pound pecans, and 1 pound cashews; can II contains 4 pounds peanuts, \(\frac{1}{2}\) pound pecans, and \(\frac{1}{2}\) pound cashews; and can III contains 5 pounds peanuts. The selling price is $28 for can I, $24 for can II, and $20 for can III. How many cans of each kind should be made to maximize revenue?

4.2.37. **Maximizing Profit:** A wood cabinet manufacturer produces cabinets for television consoles, stereo systems, and radios, each of which must be assembled, decorated, and crated. Each television console requires 3 hours to assemble, 5 hours to decorate, and 0.1 hour to crate and returns a profit of $10. Each system requires 10 hours to assemble, 8 hours to decorate, and 0.6 hour to crate and returns a profit of $25. Each radio requires 1 hour to assemble, 1 hour to decorate, and 0.1 hour to crate and returns a profit of $3. The manufacturer has 30,000, 40,000, and 120 hours available weekly for assembling, decorating, and crating, respectively. How many units of each product should be manufactured to maximize profit?

4.2.39. **Maximizing Profit:** A large TV manufacturer has warehouse facilities for storing its 25-inch color TVs in Chicago, New York, and Denver. Each month the city of Atlanta is shipped at most four hundred 25-inch TVs. The cost of transporting each TV to Atlanta from Chicago, New York, and Denver averages $20, $20, and $40, respectively, while the cost of labor required for packing averages $6, $8, and $4,
respectively. Suppose $10,000 is allocated each month for transportation costs and $3000 is allocated for labor costs. If the profit on each TV made in Chicago is $50, in New York is $80, and in Denver is $40, how should monthly shipping arrangements be scheduled to maximize profit?

Homework 22

(In Problems 1-6 determine which of the given minimum problems are in standard form.)

4.3.1. Minimize

\[ C = 2x_1 + 3x_2 \]

Subject to the constraints

\[ 4x_1 - x_2 \geq 2 \]
\[ x_1 + x_2 \geq 1 \]
\[ x_1 \geq 0 \quad x_2 \geq 0 \]

4.3.3. Minimize

\[ C = 2x_1 - x_2 \]

Subject to constraints

\[ 2x_1 - x_2 \geq 1 \]
\[ -2x_1 \geq -3 \]
\[ x_1 \geq 0 \quad x_2 \geq 0 \]

4.3.5. Minimize

\[ C = 3x_1 + 7x_2 + x_3 \]

Subject to constraints

\[ x_1 + x_3 \leq 6 \]
\[ 2x_1 + x_2 \geq 4 \]
\[ x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \]

For questions 9 and 11 write the dual problem of each minimum problem

4.3.9. Minimize

\[ C = 3x_1 + x_2 + x_3 \]

Subject to

\[ x_1 + x_2 + x_3 \geq 5 \]
\[ 2x_1 + x_2 \geq 4 \]
\[ x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \]

4.3.11. Minimize
\[ C = 3x_1 + 4x_2 + x_3 + 2x_4 \]

Subject to
\[ x_1 + x_2 + x_3 + 2x_4 \geq 60 \]
\[ 3x_1 + 2x_2 + x_3 + 2x_4 \geq 90 \]
\[ x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0, \quad x_4 \geq 0 \]

**Solve each minimum problem using the duality principle**

4.3.15. Minimize
\[ C = 6x_1 + 3x_2 \]
Subject to
\[ x_1 + x_2 \geq 4 \]
\[ 3x_1 + 4x_2 \geq 12 \]
\[ x_1 \geq 0, \quad x_2 \geq 0 \]

4.3.17. Minimize
\[ C = x_1 + 2x_2 + x_3 \]
Subject to
\[ x_1 - 3x_2 + 4x_3 \geq 12 \]
\[ 3x_1 + x_2 + 2x_3 \geq 10 \]
\[ x_1 - x_2 - x_3 \geq -8 \]
\[ x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \]

4.3.21. **Diet Preparation:** Mr. Jones needs to supplement his diet with at least 50 mg calcium and 8 mg iron daily. The minerals are available in two types of vitamin pills, P and Q. Pill P contains 5 mg calcium and 2 mg iron, while Pill Q contains 10 mg calcium and 1 mg iron. If each P pill costs 3 cents and each Q pill costs 4 cents, how could Mr. Jones minimize the cost of adding the minerals to his diet? What would the daily minimum cost be?

4.3.22. **Production Schedule:** A company owns two mines. Mine A produces 1 ton of high-grade ore, 3 tons of medium-grade ore, and 5 tons of low-grade ore each day. Mine B produces 2 tons of each grade ore per day. The company needs at least 80 tons of high-grade ore, at least 160 tons of medium-grade ore, and at least 200 tons of low-grade ore. How many days should each mine be operated to minimize costs if it costs $2000 per day to operate each mine?

4.3.23. **Production Schedule:** Argus Company makes three products: A, B, and C. Each unit of A costs $4, each unit of B costs $2, and each unit of C costs $1 to produce. Argus must produce at least 20 As, 30 Bs, and 40 Cs, and cannot produce fewer than 200 total units of As, Bs, and Cs combined. Minimize Argus’s costs.
4.3.26. **Inventory Control:** A department store stocks three brands of toys: A, B, and C. Each unit of brand A occupies 1 square foot of shelf space, each unit of brand B occupies 2 square feet, and each unit of brand C occupies 3 square feet. The store has 120 square feet available for storage. Surveys show that the store should have one hand at least 12 units of brand A and at least 30 units of A and B combined. Each unit of brand A costs the store $8, each unit of brand B $6, and each unit of brand C $10. Minimize the cost to the store.

Homework 23: Test 3 Handout

Homework 24: Test 1 and 2 Review Handout