Homework 13

8.3.10. **Raffles:** In a raffle 1000 tickets are being sold at $1.00 each. The first prize is $100. There are 2 second prizes of $50 each, and 5 third prizes of $10 each (there are 8 prizes in all). Jenny buys 1 ticket. How much more than the expected value of the ticket does she pay?

8.3.11. A fair coin is tossed 3 times, and a player wins $3 if 3 tails occur, wins $2 if 2 tails occur, and loses $3 if no tails occur. If 1 tail occurs, no one wins.
   (a) What is the expected value of the game?
   (b) Is the game fair?
   (c) If the answer to part (b) is “No,” how much should the player win or lose for a toss of exactly 1 tail to make the game fair?

8.3.12. Colleen bets $1 on a 2-digit number. She wins $75 if she draws her number from the set of all 2-digit numbers. {00, 01, 02, . . ., 99}; otherwise, she loses her $1.
   (a) Is this game fair to the player?
   (b) How much is Colleen expected to lose in a game?

8.3.13. Two teams have played each other 14 times. Team A won 9 games, and team B won 5 games. They will play again next week. Bob offers to bet $6 on team A while you bet $4 on team B. The winner gets the $10. Is the bet fair to you in view of the past records of the two teams? Explain you answer.

8.3.14. A department store wants to sell 11 purses that cost the store $40 each and 32 purses that cost the store $10 each. If all purses are wrapped in 43 identical boxes and if each customer picks a box randomly, find
   (a) Each customer’s expectation.
   (b) The department store’s expected profit if it charges $15 for each box.

8.3.15. Sarah draws a card from a deck of 52 cards. She receives 40 cents for a heart, 50 cents for an ace, and 90 cents for the ace of hearts. If the cost of a draw is 15 cents, should she play the game? Explain.

8.3.16. **Family Size:** The following data give information about family size in the United States for a household containing a husband and wife where the husband is in the 30-34 age bracket:

<table>
<thead>
<tr>
<th>Number of Children</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of Families</td>
<td>10.2%</td>
<td>15.9%</td>
<td>31.8%</td>
<td>42.1%</td>
</tr>
</tbody>
</table>

A family is chosen at random. Find the expected number of children in the family.
(In problems 33-40 find the maximum and minimum values (if possible) of the given objective function subject to the constraints.)

\[
\begin{align*}
  x + y & \leq 10 \\
  2x + y & \geq 10 \\
  x + 2y & \geq 10 \\
  x & \geq 0 \\
  y & \geq 0
\end{align*}
\]

3.2.35. \( z = 5x + 2y \)

3.3.1. **Optimal Land Use:** A farmer has 70 acres of land available on which to grow some soybeans and some corn. The cost of cultivation per acre, the workdays needed per acre, and the profit per acre are indicated in the table:

<table>
<thead>
<tr>
<th></th>
<th>Soybeans</th>
<th>Corn</th>
<th>Total Available</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cultivation Cost</strong></td>
<td>$60</td>
<td>$30</td>
<td>$1800</td>
</tr>
<tr>
<td><strong>Per Acre</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Days of Work</strong></td>
<td>3 days</td>
<td>4 days</td>
<td>120 days</td>
</tr>
<tr>
<td><strong>Per Acre</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Profit per Acre</strong></td>
<td>$300</td>
<td>$150</td>
<td></td>
</tr>
</tbody>
</table>

As indicated in the last column, the acreage to be cultivated is limited by the amount of money available for cultivation costs and by the number of working days that can be put into this part of the business. Find the number of acres of each crop that should be planted in order to maximize the profit.

3.3.2. **Investment Strategy:** An investment broker wants to invest up to $20,000. She can purchase a type A bond yielding a 10% return on the amount invested and she can purchase a type B bond yielding a 15% return on the amount invested. She also wants to invest at least as much in the type A bond as in the type B bond. She will also invest at least $5000 in the type A bond and no more than $8000 in the type B bond. How much should she invest in each type of bond to maximize her return?

3.3.3. **Manufacturing:** A factory manufactures two products, each requiring the use of three machines. The first machine can be used at most 70 hours; the second machine at most 40 hours; and the third machine at most 90 hours. The first product requires 2 hours on machine 1, 1 hour on machine 2, and 1 hour on machine 3; the second product requires 1 hour each on machines 1 and 2, and 3 hours on machine 3. If the profit is $40 per unit for the first product and $60 per unit for the second product, how many units of each product should be manufactured to maximize profit?

**Homework 14**
(In Problems 1-12 graph each inequality)

3.1.6. \( x \geq 2 \)

3.1.10. \( x + 2y > 4 \)

3.1.14. Without graphing, determine which of the points \( P_1 = (9, -5), P_2 = (12, -4), P_3 = (4,1) \) are part of the graph of the following system:

\[
\begin{align*}
10x + y &\leq 0 \\
-x + 2y &\geq 0 \\
x + y &\leq 15
\end{align*}
\]

(In Problems 27-38 graph each system of linear inequalities. Tell whether the graph is bounded or unbounded and list each corner point of the graph.)

3.1.29. \[
\begin{align*}
x + y &\geq 2 \\
2x + 3y &\leq 6 \\
x &\geq 0 \\
y &\geq 0
\end{align*}
\]

3.1.31. \[
\begin{align*}
2 &\leq x + y \\
x + y &\leq 8 \\
2x + y &\leq 10 \\
x &\geq 0 \\
y &\geq 0
\end{align*}
\]

(In Problems 1-10 the given figure illustrates the graph of the set of feasible points of a linear programming problem. Find the maximum and minimum values of each objective functions.)

3.2.1. \( z = 2x + 3y \) \( \text{(Use the graphic below)} \)
(In Problems 17-24 maximize (if possible) the quantity \( z = 5x + 7y \) subject to the given constraints.)

\[
\begin{align*}
3.2.19. & \quad \begin{cases} 
x + y & \geq 2 \\
2x + 3y & \leq 6 \\
x & \geq 0 \\
y & \geq 0
\end{cases}
\end{align*}
\]

3.3.4. **Diet:** A diet is to contain at least 400 units of vitamins, 500 units of minerals, and 1400 calories. Two foods are available: \( F_1 \), which costs $0.05 per unit, and \( F_2 \), which costs $0.03 per unit. A unit of food \( F_1 \) contains 2 units of vitamins, 1 unit of minerals, and 4 calories; a unit of food \( F_2 \) contains 1 unit of vitamins, 2 units of minerals, and 4 calories. Find the minimum cost for a diet that consists of a mixture of these two foods and also meets the minimal nutrition requirements.

3.3.5. **Investment Strategy:** A financial consultant wishes to invest up to a total of $30,000 in two types of securities, one that yields 10% per year and another that yields 8% per year. Furthermore, she believes that the amount invested in the first security should be at most one-third of the amount invested in the second security. What investment program should the consultant pursue in order to maximize income?

3.3.6. **Scheduling:** Blink appliances has a sale on microwaves and stoves. Each microwave requires 2 hours to unpack and set up, and each stove requires 1 hour. The storeroom space is limited to 50 items. The budget of the store allows only 80 hours of
employee time for unpacking and setup. Microwaves sell for $300 each, and stoves sell for $200 each. How many of each should the store order to maximize revenue?

3.3.7. **Cost Control:** An appliance repair shop has 5 vacuum cleaners, 12 TV sets, and 18 VCRs to be repaired. The store employs two part-time repairmen. One repairman can repair one vacuum cleaner, three TV sets, and three VCRs in 1 week, while the second repairmen can repair one vacuum cleaner, two TV sets and six VCRs in 1 week. The first employee is paid $250 a week and the second employee is paid $220 a week. To minimize the cost, how many weeks should each of the two repairmen be employed?

**Homework 15**

(In Problems 29-52 solve each system of equations by finding the reduced row-echelon form of the augmented matrix. If there is no solution, say the system is inconsistent.)

2.3.29. \[
\begin{align*}
    x + y &= 3 \\
    2x - y &= 3 \\
    x - y &= 1
\end{align*}
\]

2.3.37. \[
\begin{align*}
    y - z &= 6 \\
    x + z &= -1 \\
    x + 2x_2 + 3x_3 - x_4 &= 0 \\
    3x_1 - x_4 &= 4 \\
    x_2 - x_3 - x_4 &= 2
\end{align*}
\]

2.3.41. \[
\begin{align*}
    x + y + z &= 3 \\
    x - y + z &= 7 \\
    x - y - z &= 1
\end{align*}
\]

2.3.44. \[
\begin{align*}
    3x - y + 2z &= 3 \\
    3x + 3y + z &= 3 \\
    3x - 5y + 3z &= 12 \\
    x + y + z &= 3
\end{align*}
\]

2.3.45. \[
\begin{align*}
    2x + y + z &= 0 \\
    3x + y + z &= 1
\end{align*}
\]
2.3.51. \[
\begin{align*}
2x - y + z &= 6 \\
3x - y + z &= 6 \\
4x - 2y + 2z &= 12
\end{align*}
\]

2.3.52. \[
\begin{align*}
x - y + z &= 2 \\
2x - 3y + z &= 0 \\
3x - 3y + 3z &= 6
\end{align*}
\]

2.3.53. **Mixing Chemicals:** A chemistry laboratory has available three kinds of hydrochloric acid (HCl): 10%, 30%, and 50% solutions. How many liters of each should be mixed to obtain 100 liters of 25% HCl? Provide a table showing at least six of the possible solutions.

2.3.54. Repeat Problem 53 if the mixture is to be 100 liters of 40% HCl.

**Homework 16: Dependent System Handout**

**Homework 17**

(In Problems 1-12 tell whether the given matrix is in reduced row-echelon form.)

2.3.2. \[
\begin{bmatrix}
1 & 2 & 3 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

2.3.3. \[
\begin{bmatrix}
1 & 1 & 0 \\
0 & 1 & 0
\end{bmatrix}
\]

2.3.4. \[
\begin{bmatrix}
1 & 0 & 3 \\
0 & 1 & 0
\end{bmatrix}
\]

2.3.5. \[
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]

2.3.6. \[
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
2.3.7. \[
\begin{bmatrix}
1 & 2 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]

2.3.8. \[
\begin{bmatrix}
1 & 0 & 8 \\
0 & 2 & 9
\end{bmatrix}
\]

2.3.9. \[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

2.3.10. \[
\begin{bmatrix}
1 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}
\]

2.3.11. \[
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 2 \\
0 & 0 & 0
\end{bmatrix}
\]

2.3.12. \[
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & 2
\end{bmatrix}
\]

(In Problems 13-28 the reduced row-echelon form of the augmented matrix of a system of linear equations is given. Tell whether the system has one solution, no solution, or infinitely many solutions.)

2.3.13. \[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]

2.3.14. \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

2.3.15. \[
\begin{bmatrix}
1 & 0 & 4 \\
0 & 1 & 5
\end{bmatrix}
\]

2.3.16. \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 6
\end{bmatrix}
\]

2.3.17. \[
\begin{bmatrix}
1 & 0 & -2 & 6 \\
0 & 1 & 3 & 1
\end{bmatrix}
\]
<table>
<thead>
<tr>
<th>2.3.18</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>2</th>
<th>0</th>
<th>1</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
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<td>0</td>
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<td>0</td>
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<table>
<thead>
<tr>
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<th>2</th>
<th>0</th>
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<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2.3.21</th>
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<th>0</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
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<td>0</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>2.3.22</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>2.3.23</th>
<th>1</th>
<th>0</th>
<th>0</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>2.3.24</th>
<th>1</th>
<th>2</th>
<th>0</th>
<th>0</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
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<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tbody>
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<table>
<thead>
<tr>
<th>2.3.25</th>
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<th>0</th>
<th>1</th>
<th>0</th>
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<tbody>
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<td>1</td>
<td>2</td>
<td>1</td>
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</table>

<table>
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<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
2.3.27. \[
\begin{bmatrix}
1 & 0 & 0 & -1 \\
0 & 1 & 2 & 3
\end{bmatrix}
\begin{bmatrix}
0
\end{bmatrix}
\]

(In Problems 1-12 write the dimension of each matrix.)

2.4.3. \[
\begin{bmatrix}
2 & 1 & -3 \\
1 & 0 & -1
\end{bmatrix}
\]

2.4.10. \[
2 \ 1 \ -3
\]

(In Problems 13-24 determine whether the given statements are true or false, tell why.)

2.4.13. \[
\begin{bmatrix}
0 \\
1
\end{bmatrix}
= 0 \ 1
\]

2.4.14. \[
\begin{bmatrix}
3 & 2 \\
-1 & 0
\end{bmatrix}
= \begin{bmatrix}
3 & 2 \\
-1 & 4
\end{bmatrix}
\]

2.4.15. \[
\begin{bmatrix}
5 & 0 \\
0 & 1
\end{bmatrix}
\text{ is square}
\]

2.4.16. \[
\begin{bmatrix}
3 & 2 & 1 \\
4 & -1 & 0
\end{bmatrix}
\text{ is } 3 \times 2
\]

2.4.17. \[
\begin{bmatrix}
x & 2 \\
4 & 0
\end{bmatrix}
= \begin{bmatrix}
3 & 2 \\
4 & 0
\end{bmatrix}
\text{ if } x=3
\]

2.4.18. \[
\begin{bmatrix}
x & y \\
0 & 0
\end{bmatrix}
= x \ y
\]

2.4.19. \[
\begin{bmatrix}
5 & 0 \\
1 & 1
\end{bmatrix}
= \begin{bmatrix}
2+3 & 0 \\
1 & 1
\end{bmatrix}
\]

2.4.20. \[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
= \begin{bmatrix}
3 - 2 & 3 - 3 \\
3 - 3 & 3 - 2
\end{bmatrix}
\]

2.4.21. \[
2 \begin{bmatrix}
1 & 0 \\
0 & 2
\end{bmatrix}
= \begin{bmatrix}
2 & 0 \\
0 & 4
\end{bmatrix}
\]
2.4.22. \[
\begin{pmatrix}
8 & 0 \\
5 & -1
\end{pmatrix} = \begin{pmatrix}
-8 & 0 \\
5 & 1
\end{pmatrix}
\]

2.4.23. \[
\begin{pmatrix}
8 \\
1
\end{pmatrix} + \begin{pmatrix}
2 \\
9
\end{pmatrix} = 10
\]

(In Problems 33-50 use the matrices below. For Problems 33-44 perform the indicated operation(s); for Problems 45-50 verify the indicated property.)

\[
A = \begin{pmatrix}
2 & -3 & 4 \\
0 & 2 & 1
\end{pmatrix} \quad B = \begin{pmatrix}
1 & -2 & 0 \\
5 & 1 & 2
\end{pmatrix} \quad C = \begin{pmatrix}
-3 & 0 & 5 \\
2 & 1 & 3
\end{pmatrix}
\]

2.4.35. \(2A - 3C\)

2.4.42. \(2A - 5(B + C)\)

Homework 18: Test II Handout