Problem 1. The acceleration of a moving object is given by \( \vec{a}(t) = (12t, 2, 6t + 4) \). Find the position vector of this object with the initial conditions \( \vec{v}(0) = (1, 2, 1) \) and \( \vec{r}(0) = (1, -2, -1) \).

Problem 2. The acceleration of a moving object is given by \( \vec{a}(t) = (6, 4 - 12t, 24t) \). Find the position vector of this object such that \( \vec{v}(1) = (1, 2, 1) \) and \( \vec{r}(2) = (0, 2, -1) \).

Problem 3. Find the domain of \( f(x, y) = \sqrt{4 - x^2 - y^2} + \ln (x^2 y) \). Then graph this domain.

Problem 4. Find the domain of \( f(x, y) = \sqrt{9 - x^2 - y^2} + \arcsin (x) \). Then graph this domain.

Problem 5. Find the domain of \( f(x, y) = \sqrt{x^2 + y^2 - 4} + \ln(4 - y - x) \). Then graph this domain.

Problem 6. Show that \( \lim_{(x,y) \to (0,0)} \frac{xy^3}{x^6 + y^2} = 0 \).

Problem 7. Show that the following limit does not exist: \( \lim_{(x,y) \to (0,0)} \frac{x^2 y}{x^4 + y^2} \).

Problem 8. Show that the following limit does not exist: \( \lim_{(x,y) \to (0,0)} \frac{x \sqrt{y}}{x^2 + y} \).

Problem 9. Find \( \frac{\partial^2 f}{\partial x \partial y} \), and \( \frac{\partial^2 f}{\partial y \partial x} \) when \( f(x, y) = x \sin y + \int_y^{2x} \sin(t^2) \, dt \).

Problem 10. Find \( \frac{\partial^2 f}{\partial x \partial y} \), and \( \frac{\partial^2 f}{\partial y \partial x} \) when \( f(x, y) = x^2 \cos y + \int_{2y}^{xy} \sqrt{1 + t^4} \, dt \).

Problem 11. Show that the function \( f(x, y) = x^3 - 3xy^2 + e^{3x} \cos 3y \) is harmonic.

Problem 12. Find \( g'(t) \) when \( g(t) = f(4 \sin 2t, 5 \cos 2t) \).

Problem 13. Use differential (linear approximation) to estimate \( \sqrt{16.03} \cdot \sin \left( \frac{7\pi}{20} \right) \).

Problem 14. Use differential (linear approximation) to estimate \( \sqrt{81.05} \cdot \sin \left( \frac{21\pi}{80} \right) \).

Problem 15. Let \( f(x, y) = \sin x \cos y \). Use the linear approximation to estimate \( f \left( \frac{9\pi}{20}, \frac{3\pi}{10} \right) \).

Problem 16. Find the direction in which \( f(x, y, z) = e^x + e^y - e^{2z} \) increases most rapidly at \( (1, 1, -1) \).
Problem 17. Find the directional derivative of $f(x, y, z) = x^2 + yz$ at $(1, -3, 2)$ in the direction of the path $r(t) = t^2 \mathbf{i} + 3t \mathbf{j} + (1 - t^3) \mathbf{k}$

Problem 18. Use the chain rule to find $\frac{du}{dt}$ where $u = x^2 - 3xy + 2y^2; \ x = \cos t, \ y = \sin t$.

Problem 19. Use the chain rule to find $\frac{\partial u}{\partial t}, \frac{\partial u}{\partial s}$ for $u = x^2 - 3xy + 2y^2; \ x = s \cdot \cos t, \ y = t \cdot \sin s$.

Problem 20. Write equation of tangent plane to the surface $z = 3x^2y - xy^3$ at the point $P = (-1, 1, 4)$ of this surface.

Problem 21. Find an equation for the tangent plane and symmetric equations for the normal line to the surface $x^2z + xy^2 - z^3 = 5$ at the point $(2, -1, 1)$.

Problem 22. Find the critical points of $f(x, y) = x^3 + y^3 - 3x^2 + 6y^2 + 3x + 12y + 7$, and test them for local maximum and minimum.

Problem 23. Find the absolute maximum and minimum of the function $f(x, y) = x^2 + y^2 + x^2y + 4$ on the set $\{(x, y) : |x| \leq 1, |y| \leq 1\}$.

Problem 24. Find the absolute maximum and minimum of the function $f(x, y) = 2x - 2xy + y^2$ over the region in $xy$-plane bounded by the graphs of $y = x^2, y = 2$.

Problem 25. Find the extreme values of $f(x, y) = x^2 + 2y^2 - 2x + 3$ subject to the constraint $x^2 + y^2 \leq 10$.

Problem 26. Evaluate the integral $\int \int_R (4y + x) \, dA$, where $R$ is the region between the parabola $y = 3 - x^2$ and $y = 2x$.

Problem 27. Evaluate $\int \int_R (4 + x^2) \, dA$, where $R$ is the region between the parabolas $y = 1 + x^2$, and $y = 3 - x^2$.

Problem 28. Evaluate $\int \int_R xy \, dA$, where $R$ is the region in the first quadrant of $xy$-plane bounded by the graph of $y = x^2 + x$ and $y = x^2 - x$ and $y = 2$.

Problem 29. Evaluate $\int_0^2 \int_{x^2}^4 x \, e^{y^2} \, dy \, dx$

Problem 30. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{(x^2+y^2)} \, dy \, dx$.

Problem 31. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} \sin(x^2 + y^2) \, dx \, dy$. 