The sequence satisfies a *linear recurrence relation of order* $k$ if there exist quantities with $\neq 0$ and a quantity such that for $n \geq k$

Note: The quantities may be constants or functions of $n$.

Note: The linear recurrence relation is called *homogeneous* if

Note: The linear recurrence relation is said to have *constant coefficients* if

Example: Fibonacci numbers

THEOREM: Let be a nonzero number. Then $h_n =$ is a solution of the linear homogeneous recurrence relation

$\quad (\neq 0 \text{ and } n \geq k)$

with constant coefficients if and only if is a root of the polynomial equation

If the polynomial equation has $k$ distinct roots, then

is the general solution of the recurrence relation in the following sense: No matter what initial values for are given, there are constants so that

is the unique solution which satisfies both the recurrence relation and the initial values.

Example: Solve the recurrence relation $h_n = 2h_{n-1} + 2h_{n-2}$ $(n \geq 2)$ subject to $h_0 = 1$ and $h_1 = 3$.
Example: Solve the recurrence relation \( h_n = 5h_{n-1} - 6h_{n-2} \) (\( n \geq 2 \)) subject to \( h_0 = 1 \) and \( h_1 = -2 \).
Example: Solve the recurrence relation $h_n = -h_{n-1} + 3h_{n-2} + 5h_{n-3} + 2h_{n-4}$ ($n \geq 4$) subject to the initial values $h_0 = 1$, $h_1 = 0$, $h_2 = 1$, and $h_3 = 2$. 