6.2 Combinations with Repetition

Recall: For $T = \{ \}$, there are $r$-combinations of $T$.

In fact an $r$-combination of $T$ has the form

where

And counting the number of $r$-combinations of $T$ is equivalent to counting the number of

Also note that if $T = \{ \} = \{ \}$ ($T$ is a)

then the number of $r$-combinations of $T$ is

Example: Determine the number of 10-combinations of $T = \{3 \cdot a, 4 \cdot b, 5 \cdot c\}$.

Solution 1 (old way)

Solution 2 (new way) We set up the solution in terms of the inclusion-exclusion principle.

Let $S$ be the set of all 10-combinations of

Let be the property that a 10-combination contains than 3 $a$'s.
Let be the property that a 10-combination contains than 4 $b$'s.
Let be the property that a 10-combination contains than 5 $c$'s.

For each $i = 1, 2, 3$, let be the 10-combinations of satisfying property

The number of 10-combinations of $T$ is , which by the inclusion-exclusion principle is
Thus our answer is

Example: What is the number of integral solutions of the equation which satisfy

Solution: We first introduce new variables so that our original question is equivalent to counting the integral solutions of the equation which satisfy
And this is equivalent to counting the number of 16-combinations of

Now we continue and put in terms of the inclusion-exclusion principle.

Let \( S \) be the set of all

Let \( A_1 \) be the property that the integral solution satisfies

Let \( A_2 \) be the property that the integral solution satisfies

Let \( A_3 \) be the property that the integral solution satisfies

Let \( A_4 \) be the property that the integral solution satisfies

And for each \( i = 1, 2, 3, 4 \), let \( A_i \) be the set of integral solutions satisfying property \( A_i \).

Then we want

\[ |S| = \]

\[ |A_1| = \]

\[ |A_2| = \]

\[ |A_3| = \]

\[ |A_4| = \]

\[ |A_1 \cap A_2| = \]

\[ |A_1 \cap A_3| = \]

\[ |A_1 \cap A_4| = \]

\[ |A_2 \cap A_3| = \]

\[ |A_2 \cap A_4| = \]

\[ |A_3 \cap A_4| = \]

\[ |A_1 \cap A_2 \cap A_3| = \]

\[ |A_1 \cap A_2 \cap A_4| = \]

\[ |A_1 \cap A_3 \cap A_4| = \]

\[ |A_2 \cap A_3 \cap A_4| = \]