3.1 & 3.2 Pigeonhole Principle (simple and strong forms)

Pigeonhole Principle

Proof

In terms of colors: If $n + 1$ objects are colored with $n$ colors, then two objects have

Example: Among 13 people, two have their birthday in the same month.

Example: With $n$ married couples, how many of the $2n$ people must be selected in order to guarantee that a married couple is chosen?

Example: Given integers $a$ and $b$, there exist integers $w$ with $a < w < b$ such that

that is,

Why is this true?
Strong Form of Pigeonhole Principle

Proof

Simple form is the special case when

Example: A basket of fruit is being arranged out of apples, bananas, and oranges. What is the smallest number of pieces of fruit that should be put in the basket to guarantee that either there are at least eight apples or at least six bananas or at least nine oranges?
Corollary  Let $n$ and $r$ be positive integers. If objects are distributed into $n$ boxes, then at least one of the boxes contains $r$ or more of the objects.

Averaging Principle  If the average of $n$ nonnegative integers is greater than that is,

then at least one of the integers is greater than or equal to

Proof