2.5 Combinations of Multisets

Let $S$ be a multiset. An $r$-combination of $S$ is

( an unordered selection of $r$ elements from $S$)

Example: Find the 3-combinations of $S = \{2 \cdot a, 1 \cdot b, 3 \cdot c\}$

Example: How many 3-combinations of $S = \{\infty \cdot a, \infty \cdot b, \infty \cdot c\}$ are there?
THEOREM  Let $S$ be a multiset with objects of 

, each with 

Then the number of 

is 

PROOF  Let $S = \{ \}$. An $r$-combination of $S$ has the form 

\{ \}, where each 

and 

Thus the number of 

equals the number of 

We show that the number of such 

that is, we establish a 

First, suppose that 

is a particular solution with 

Then define the $(r + k - 1)$-permutation 

where, of course, if 

, then there are no 1’s between the 

Notice that there are 1’s and ’s and so we have an 

On the other hand, let us begin with an 

then define 

to be the number 

define 

to be the number 

and for each $i = 2, 3, \ldots, k - 1$, define 

to be the number 

In this way, 

will be nonnegative integers with 

So we have our desired one-to-one correspondence and so the number of 

is the number of 

which is
Example: Bakery sells eight varieties of bagels. If a box of bagels contains one dozen, how many different options are there for a box of bagels? (assume unlimited supply of each type of bagel)

Example: Same bakery as previous example, but what if we insist that there is at least one of each type of bagel? (How many different options are there for a box of bagels?)

Example: How many integral solutions of the equation in which ?