11.7 More on Trees
We will cover several algorithms for finding a spanning tree in a connected graph.

Algorithm to grow a spanning tree
Let $G$ be a graph of order $n$ and let $v$ be a vertex of $G$.
(1) Put $V$ and $E$.
(2) While there exists a vertex $v'$ in $V$ and a vertex $v''$ not in $V$ such that $(v', v'')$ is an edge of $G$.
   (i) Put the vertex $v'$ in $V$.
   (ii) Put the edge $(v', v'')$ in $E$.
(3) Put $V$.

THEOREM Let $G$ be a graph. Then $G$ is connected if and only if the graph constructed by carrying out the algorithm of $G$ is a spanning tree of $G$.

Proof

Breadth-first algorithm $bf(x)$ denotes the
Let $G$ be a graph of order $n$ and let $v$ be a vertex of $G$.
(1) Put $V$ and $E$.
(2) If there is no edge in $E$ that joins a vertex in $V$ to a vertex not in $V$, then stop.
   Otherwise, find an edge with a vertex in $V$ and a vertex not in $V$ such that has smallest breadth-first number, and do the following
   (i) Put $V$.
   (ii) Put $E$.
   (iii) Put the vertex into $V$.
   (iv) Put the edge into $E$.
   (v) Put $V$.
   (vi) Increase by 1 and go back to (2).

The keeps track of the order that the vertex is chosen and put into $V$. 
THEOREM (BF-algorithm) Let be a graph and let be a vertex of . Then is connected if and only if the graph constructed by carrying out the BF-algorithm is a of . If is connected, then, for each vertex of , the in between and equals , and this is the same as the between and in .
Depth-first algorithm \( df(x) \) denotes the

Let \( G \) be a graph of order \( n \) and let \( v \) be a vertex of \( G \).

(1) Put

(2) If there is no edge in \( G \) that joins a vertex in \( G \) to a vertex not in \( G \), then stop.

Otherwise, find an edge \( uv \) with \( u \) in \( G \) and \( v \) not in \( G \) such that \( v \) has largest depth-first number, and do the following

(i) Put

(ii) Put the vertex \( v \) into

(iii) Put the edge \( uv \) into

(iv) Put

(v) Increase \( d_f(v) \) by 1 and go back to (2)

THEOREM (DF-algorithm) Let \( G \) be a graph and let \( v \) be a vertex of \( G \).

Then \( v \) is in a spanning tree of \( G \) if and only if the graph constructed by carrying out the DF-algorithm is a spanning tree of \( G \).