11.4 Bipartite multigraphs

A multigraph \( G \) is \( \text{if its vertex set can be partitioned into two subsets } X \text{ and } Y \) such that each edge of \( G \) joins

The pair \( X, Y \) is called a \( \text{of } G. \)

Example

Example

Special class of graphs:

Recall: The \( d(x, y) \) between two vertices and \( x \) and \( y \) is the

THEOREM A multigraph is \( \text{if and only if each of its cycles has} \)

Proof
11.5 Trees

Example

Recall: An edge is a \text{\underline{\hspace{1cm}}} if its removal leaves the graph disconnected.

THEOREM A connected graph of order \( n \) has at least \( \text{\underline{\hspace{1cm}}} \) edges. Moreover, for each positive integer \( n \), there exist connected graphs with exactly \( \text{\underline{\hspace{1cm}}} \) edges. Removing any edge from a connected graph of order \( n \) with exactly \( \text{\underline{\hspace{1cm}}} \) edges leaves a and hence each edge is a

A \( \text{\underline{\hspace{1cm}}} \) is defined to be a \( \text{\underline{\hspace{1cm}}} \) in which \( \text{\underline{\hspace{1cm}}} \) is a

THEOREM Let \( \text{\underline{\hspace{1cm}}} \) be a connected graph of order \( \text{\underline{\hspace{1cm}}} \). Then \( \text{\underline{\hspace{1cm}}} \) is a \( \text{\underline{\hspace{1cm}}} \) if and only if \( \text{\underline{\hspace{1cm}}} \) has edges.

Proof
LEMMA  Let $G$ be a connected graph and let $f$ be an edge of $G$. Then $G$ is a tree if and only if $f$ contains $f$.

Proof

THEOREM  Let $G$ be a connected graph. Then $G$ is a tree if and only if $G$ has no cycles.

Proof

THEOREM  A graph $G$ is a tree if and only if for every pair of distinct vertices $u$ and $v$, there is a unique path joining $u$ and $v$, which necessarily has length equal to the distance between $u$ and $v$.

Proof
Let $G$ be a graph. A \text{ of } $G$ is a

An edge incident with a \text{ is a }

Examples

THEOREM Let $G$ be a $\ldots$ of order $\ldots$ Then $\ldots$ has at least two

Proof

Example

A tree that is a spanning subgraph of $G$ is a $\ldots$ of $G$.

Example How can we find a spanning tree for a connected graph $G$?

THEOREM Every connected graph