11.1 Basic Properties of Graphs

graph

vertices
edges

order
size

adjacent
incident

geometric diagram of a graph
vertex-point
edge-curve

multiplicity of an edge
loop

multigraph
general graph

complete graph of order \( n \)
planar

plane-graph
planar representation of a graph

degree
deg(x)
degree sequence

THEOREM Let $G$ be a general graph with vertices and having $m$ edges. Then

Proof When summing the degrees of the vertices of a graph, each edge is counted

COROLLARY A general graph has an even number of vertices with odd degree.

Two general graphs and are isomorphic if there is a function such that for each pair of vertices $x$ and $y$ of , there are as many of joining $x$ and $y$ as there are of joining and is called an isomorphism of and

THEOREM Two isomorphic general graphs have the same degree sequence, but two graphs with the same degree sequence need not be isomorphic.
walk of length $m$
joins the vertices $x$ and $y$
closed walk
open walk
trail
path
cycle
connected
disconnected
distance between two vertices $x$ and $y$
general subgraph
induced
spanning
connected components
adjacency matrix
THEOREM  Let \( G \) and \( H \) be two general graphs. Then the following are necessary conditions for \( G \) and \( H \) to be isomorphic:

(1) If \( G \) is a graph, then so is \( H \).

(2) If \( G \) is connected, so is \( H \). Indeed, \( G \) and \( H \) have the same number of connected components.

(3) If \( G \) has a cycle of length equal to some integer \( k \), then so does \( H \).

(4) If \( G \) has an (induced) general subgraph that is a complete graph \( K_m \) of order \( m \), so does \( H \).