MATHEMATICS 181: Calculus I
Summer II Session 2013
Section 1002

EXAM 2

M. Robinette
June 28, 2013

NAME ____________________

Note: You must show all necessary work in order to receive full credit. There are 21 problems, each problem is worth 5 points, for a total of 105 possible points on this exam; however, your score will not exceed 100.

1. Differentiate the function \( f(x) = 2x^3 - 5\cos x \).

\[
 f'(x) = 6x^2 + 5\sin x
\]

2. The equation of motion of a particle is \( s = 2t^3 - 7t^2 + 4t + 3 \), where \( s \) is in meters and \( t \) is in seconds. Find

(a) the velocity and acceleration as functions of \( t \),

(b) the acceleration after 1 s, and

(c) the acceleration when the velocity is 0.

(a) \( v(t) = 6t^2 - 14t + 4 \)

\( a(t) = 12t - 14 \)

(b) \( a(1) = 12 - 14 = -2 \text{ m/s}^2 \)

(c) \( v(t) = 0 \)

\[
6t^2 - 14t + 4 = 0
\]

\[
2(3t^2 - 7t + 2) = 0
\]

\[
(3t - 1)(t - 2) = 0
\]

\( t = \frac{1}{3}, \quad t = 2 \)

\( a(\frac{1}{3}) = 12(\frac{1}{3}) - 14 = -10 \text{ m/s}^2 \)

\( a(2) = 12(2) - 14 = 10 \text{ m/s}^2 \)

3. Differentiate \( f(\theta) = \frac{\tan \theta}{1 + \sec \theta} \).

\[
f'(\theta) = \frac{(1 + \sec \theta) \sec^2 \theta - \tan \theta \sec \theta \tan \theta}{(1 + \sec \theta)^2}
\]

4. If \( f(x) = \frac{x^3}{1+x} \), find \( f''(x) \).

\[
f'(x) = \frac{(1+x) \cdot 3x^2 - x^3}{(1+x)^2} \]

\[
= \frac{3x^2 + 3x^2 - x^3}{(1+x)^2} = \frac{3x^2 + 2x^3}{(1+x)^2}
\]

5. Find the derivative of the function \( h(t) = (2t+1)^{10}(3t^2 - 1)^4 \).

\[
h'(t) = (3t^2 - 1)^4 \cdot 4(2t+1)^9 \cdot 6t + (3t^2 - 1)^4 \cdot \frac{2}{9} \cdot (2t+1) \cdot 2
\]
6. Find the derivative of the function \( y = \sin(\cot 3x) \).

\[ y' = \cos(\cot 3x) \left(-\csc^2 3x \right) \cdot 3 \]

7. Find all points on the graph of the function \( f(x) = 2 \sin x + \sin^2 x \) at which the tangent line is horizontal.

\[ f'(x) = 2 \cos x + 2 \sin x \cos x = 0 \]
\[ 2 \cos x (1 + \sin x) = 0 \]
\[ \cos x = 0 \quad \text{or} \quad \sin x = -1 \]
\[ x = \frac{\pi}{2} + 2\pi k \]
\[ x = \frac{3\pi}{2} + 2\pi k \]
\[ k \text{ any integer} \]

So answer is points

\[ \left( \frac{\pi}{2} + 2\pi k, 0 \right) \]
\[ \left( \frac{3\pi}{2} + 2\pi k, -1 \right) \]

8. Find \( dy/dx \) by implicit differentiation if \( y \sin x = x^2 + y^2 \).

\[ y \cos x + \sin x \frac{dy}{dx} = 2x + 2y \frac{dy}{dx} \]
\[ \sin x \frac{dy}{dx} - 2y \frac{dy}{dx} = 2x - y \cos x \]
\[ \frac{dy}{dx} (\sin x - 2y) = 2x - y \cos x \]
\[ \frac{dy}{dx} = \frac{2x - y \cos x}{\sin x - 2y} \]

9. Gravel is being dumped from a conveyor belt at a rate of 20 ft³/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 8 ft high?

\[ \text{Given } \frac{dV}{dt} = 20 \text{ ft}^3/\text{min} \]
\[ \frac{dh}{dt} \text{ when } h = 8 \text{ ft} \]

\[ \frac{dV}{dt} = \frac{12}{\sqrt{2} \pi} \frac{dh}{dt} \]
\[ \frac{dh}{dt} \bigg|_{h=8} = \frac{12}{2 \sqrt{2} \pi} \left( -20 \right) = \frac{20}{\sqrt{2} \pi} = \frac{5}{\pi} \text{ ft/min} \]

10. Use differentials to estimate the number \((1.99)^3\).

\[ f(x) \approx x^3 \]
\[ f'(x) \approx 3x^2 \]
\[ f(x + \Delta x) \approx f(x) + f'(x) \Delta x \]
\[ x = 2 \]
\[ x + \Delta x = 1.99 \]
\[ \frac{\Delta x}{2} = 1.99 - 2 = -0.01 \]
\[ \Delta x = -0.01 \]
\[ \frac{9}{2} \Delta x = 3 \cdot (-0.01) \]
\[ f(2 + \Delta x) \approx f(2) + \frac{9}{2} \Delta x = 8 - 0.12 = 7.88 \]
11. Find the limit \( \lim_{x \to -5} e^{x^2 - 5} = \lim_{x \to -\infty} e^x = 0 \)

\[
\lim_{x \to -5} \frac{1}{x^2 - 5} = \lim_{x \to -5} \frac{-2}{x - \frac{3}{x}} = -\infty
\]

12. Find the limit \( \lim_{x \to 2^+} \ln(x^2 - 4) = \lim_{x \to 2^+} \ln(x^2 - 4) = 0^+ \)

13. Find the limit \( \lim_{x \to 2^+} [\ln(x^2) - \ln(x + 1)] = \lim_{x \to 2^+} \ln\left(\frac{x^2}{x + 1}\right) = \lim_{x \to 2^+} \ln x = 0 \)

\[
\lim_{x \to 2^+} \frac{x^2}{x + 1} = \lim_{x \to 2^+} \frac{2}{x + 1} = \lim_{x \to 2^+} \frac{1 + \frac{2}{x}}{\frac{1}{x} + 1} = 1
\]

14. Differentiate the function \( f(x) = \csc x \ln^2 x \):

\[
f'(x) = \csc x \left( \frac{2}{x} - \ln x \right)
\]

15. Differentiate the function \( G(t) = e^{10x^2} \):

\[
G'(t) = e^{10x^2} \left( 20x \cos 2x + (1 + 2x \ln 2 + x^2) \right)
\]

16. Use logarithmic differentiation to find the derivative of the function \( y = x^{2x} \):

\[
\ln y = 2x \ln x
\]

\[
\frac{1}{y} y' = 2x \cdot \frac{1}{x} + \ln x \cdot 2
\]

\[
y' = y \left( 2 + 2 \ln x \right)
\]

\[
y' = x^{2x} \left( 2 + 2 \ln x \right)
\]
17. Find the derivative of the function \( y = (\sin^{-1} x)^2 \).

\[
y' = 2 \sin^{-1} x \cdot \frac{1}{\sqrt{1-x^2}}
\]

18. Find the limit \( \lim_{x \to -\infty} \frac{e^{2x}}{x^2} \).

\[
\lim_{x \to -\infty} \frac{e^{2x}}{x^2} = 0
\]

19. Find the limit \( \lim_{x \to 0} \frac{1 - \sin 3x}{1 + \cos 6x} \).

\[
\lim_{x \to 0} \frac{1 - \sin 3x}{1 + \cos 6x} = \frac{1}{2} \lim_{x \to 0} \frac{-\sin 3x}{\cos 6x} = \frac{1}{2} \cdot \frac{\frac{3}{2}}{1} = \frac{3}{4}
\]

20. Find the limit \( \lim_{x \to \infty} \left( x - \ln x \right) \).

\[
\lim_{x \to \infty} \left( x - \ln x \right) = \infty
\]

21. Find the limit \( \lim_{x \to 0^+} \frac{\sqrt{2}^x}{\ln x} \).

\[
\lim_{x \to 0^+} \frac{\sqrt{2}^x}{\ln x} = \lim_{x \to 0^+} \frac{2^{x/2}}{\frac{1}{x}} = \lim_{x \to 0^+} 2^{x/2} x = 0
\]