8.5 Power Series

RECALL geometric series \( \sum_{n=1}^{\infty} ar^{n-1} = \)

NOW A power series has the form

\[ \sum_{n=0}^{\infty} c_n x^n \]

EXAMPLE \( c_n = 1 \) for every \( n = 0, 1, 2, 3, \ldots \)

\[ \sum_{n=0}^{\infty} c_n x^n \]

\[ \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \text{ for } |x| < 1 \text{ (that is, } -1 < x < 1) \]

AND

\[ \sum_{n=0}^{\infty} x^n \text{ diverges for } |x| \geq 1 \text{ (that is, } x \geq 1 \text{ or } x \leq -1). \]

Power series centered at \( a: \) \[ \sum_{n=0}^{\infty} c_n (x - a)^n \]

GOAL Find all values \( x \) so that the power series is convergent.

Interval of convergence \( (a - R, a + R) \) or \[ a - R, a + R \text{ or } (,] \text{ or } [,) \]

Radius of convergence \( R \)

Possibilities:

1. \( R = 0 \)
2. \( R = \infty \)
3. There is a positive real number \( R \) such that the series converges if \( |x - a| < R \) and diverges if \( |x - a| > R \). (what happens at \( |x - a| = R \) must be analyzed as special cases)
EXAMPLES    Find the radius of convergence and interval of convergence for each series.

1. \[ \sum_{n=0}^{\infty} \frac{(x - 2)^n}{\sqrt{n+1}} \]

2. \[ \sum_{n=0}^{\infty} \frac{(x - 1)^n}{n!} \]
3. \[ \sum_{n=0}^{\infty} \frac{(x - 12)^n}{2^n} \]

4. \[ \sum_{n=0}^{\infty} \frac{2^n (x - 3)^n}{n + 3} \]

5. \[ \sum_{n=1}^{\infty} \frac{(-1)^n (x + 2)^n}{n2^n} \]