5.4 The Fundamental Theorem of Calculus

FUNDAMENTAL THEOREM OF CALCULUS (Part 1) If \( f \) is continuous on \([a, b]\), then the function \( g \) defined by

\[
g(x) = \int_a^x f(t) \, dt \quad \text{for} \quad a \leq x \leq b
\]

is an antiderivative of \( f \), that is, \( g'(x) = f(x) \) for \( a < x < b \).

Another way to say this:

\[
\frac{d}{dx} \int_a^x f(t) \, dt =
\]

EXAMPLE Sketch the area represented by \( g(x) = \int_0^x (1 + \sqrt{t}) \, dt \).

Then find \( g'(x) \) in two ways:

(a) by using Part 1 of the Fundamental Theorem of Calculus

(b) by evaluating the integral using Part 2 and then differentiating
EXAMPLES  Find the derivative of each function.

1.  \[ g(x) = \int_0^x 6t^3 \, dt \]

2.  \[ h(x) = \int_0^{3x} t \, dt \]

3.  \[ y = \int_{\sin x}^{\cos x} (1 + v^2)^{10} \, dv \]