4.5 Optimization problems

STEP 1: Introduce variables and find the function that is to be minimized or maximized.
STEP 2: Find the critical numbers of the function.
STEP 3: Test the critical numbers in the second derivative test.
STEP 4: Report the answer.

EXAMPLES
1. Find a positive number such that the sum of the number and its reciprocal is as small as possible.

2. Find the dimensions of a rectangle with area 1000 m$^2$ whose perimeter is as small as possible.

3. A box with a square base and open top must have a volume of 32,000 cm$^3$. Find the dimensions of the box that minimizes the amount of material used.
4. Find the point on the graph of \( f(x) = \sqrt{x} \) that is closest to \((4, 0)\).

5. Find the dimensions of the rectangle of largest area that has its base on the \(x\)-axis and its other two vertices above the \(x\)-axis and lying on the parabola \( y = 8 - x^2 \).

6. A box with an open top is to be constructed from a 2 ft by 3 ft rectangular piece of cardboard by cutting out a square from each corner and then folding up the sides. Let \( x \) denote the length of a side of the square to be removed. Find \( x \) to maximize the volume.