3.5 Inverse trigonometric functions

RECALL: \( y = \sin^{-1} x \) means that \( y \) is the angle between \( -\frac{\pi}{2} \) and \( \frac{\pi}{2} \) with \( \sin y = x \)

Note: \( \sin^{-1} x = \arcsin x \)

FACTS: \( \sin(\sin^{-1} x) = x \) for all \( x \) in the interval \( [-1, 1] \)
\( \sin^{-1}(\sin x) = x \) for all \( x \) in the interval \( \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \)

EXAMPLE Find the exact value of \( \sin^{-1}(-1/2) \).

RECALL: \( y = \cos^{-1} x \) means that \( y \) is the angle between 0 and \( \pi \) with \( \cos y = x \)

FACTS: \( \cos(\cos^{-1} x) = x \) for all \( x \) in the interval \( [-1, 1] \)
\( \cos^{-1}(\cos x) = x \) for all \( x \) in the interval \( [0, \pi] \)

EXAMPLE Simplify the expression \( \tan(\cos^{-1} x) \)

RECALL: \( y = \tan^{-1} x \) means that \( y \) is the angle between \( -\frac{\pi}{2} \) and \( \frac{\pi}{2} \) with \( \tan y = x \)

EXAMPLE Find the exact value of \( \tan^{-1}(\tan(\frac{4\pi}{3})) \)
DERIVATIVES of Inverse Trig Functions

RECALL: \( y = \sin^{-1} x \) means that \( \sin y = x \) and \( -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \)

\[
\frac{d}{dx} \sin^{-1} x = \quad \frac{d}{dx} \cos^{-1} x = \\
\frac{d}{dx} \tan^{-1} x = \quad \frac{d}{dx} \cot^{-1} x = \\
\frac{d}{dx} \sec^{-1} x = \quad \frac{d}{dx} \csc^{-1} x = 
\]

EXAMPLES  Differentiate each function and simplify where possible.

1. \( y = (\sin^{-1} x)^2 \)

2. \( y = \tan^{-1}(x - \sqrt{1 + x^2}) \)

3. \( y = x \cos^{-1} x - \sqrt{1 - x^2} \)
4. \( f(x) = x \ln(\arctan x) \)

5. \( h(t) = e^{\sec^{-1} t} \)

**EXAMPLE** Find the limit \( \lim_{x \to 0^+} \tan^{-1}(\ln x) \)

**EXAMPLE** Find \( y' \) if \( \tan^{-1}(xy) = 1 + x^2y \)