1. (4 pts.) Find the domain and range of the function \( f(x) = \sqrt{x^2 - 3} \).

\[
\begin{align*}
\text{Domain: } & (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty) \\
\text{Range: } & [0, \infty) \\
\end{align*}
\]

2. (30 pts.) Evaluate the limit, if it exists. If the limit does not exist, explain why.

(a) \( \lim_{x \to 0} \frac{\sin 2x}{\sin 3x} = \frac{2 \lim_{x \to 0} \sin 2x}{3 \lim_{x \to 0} \sin 3x} = \frac{2}{3} \cdot 1 = \frac{2}{3} \)

(b) \( \lim_{x \to 2} \frac{x^2 - 3x + 2}{2x^2 - 3x - 2} = \lim_{x \to 2} \frac{(x-2)(x-1)}{(x-2)(2x+1)} = \frac{x-1}{2x+1} \rightarrow \frac{1}{5} \)

(c) \( \lim_{x \to 2} \frac{|x-2|}{2-x} \) does not exist since right and left hand limits are not equal.

(d) \( \lim_{x \to \infty} \frac{5+2x+x^2-2x^3}{3x^3+3x^3-2} = \lim_{x \to \infty} \frac{5}{3} \cdot \frac{\frac{1}{x^2} + \frac{1}{x} + \frac{1}{x^3} - 2}{1 + \frac{3}{x^2} - \frac{2}{x^3}} = \frac{-2}{3} \)

(e) \( \lim_{x \to 1} \frac{\sqrt{x+3} - 2}{x-1} = \lim_{x \to 1} \frac{(\sqrt{x+3} + 2)(x+3 - 4)}{(x-1)(\sqrt{x+3} + 2)} = \lim_{x \to 1} \frac{x+3 - 4}{(x-1)(\sqrt{x+3} + 2)} = \frac{1}{4} \)
3. (6 pts.) Determine (with explanation) the infinite limit \( \lim_{x \to 3^+} \frac{x^2 + 4}{(x-2)(x+3)} \). Does not exist, since
\[
\lim_{x \to 3^+} x^2 + 4 = 25 \neq 0 \quad \text{and} \quad \lim_{x \to 3^+} (x-2)(x+3) = 15.
\]

4. (6 pts.) Use the Squeeze Theorem to find \( \lim_{x \to 0^+} x^2 \cos \frac{1}{x} \).

For \( x \neq 0 \), we know \(-1 \leq \cos \frac{1}{x} \leq 1\). Multiplying through by \( x^2 \), which is positive, we obtain \(-x^2 \leq x^2 \cos \frac{1}{x} \leq x^2\). Observe that \( \lim_{x \to 0^+} -x^2 = \lim_{x \to 0^+} x^2 = 0 \).

Thus, by the Squeeze Theorem, \( \lim_{x \to 0^+} x^2 \cos \frac{1}{x} = 0 \).

5. (12 pts.) Consider the function \( f(x) = \begin{cases} 
2-x & x < -1 \\
x^2 + 2 & -1 \leq x < 1 \\
3x+1 & x \geq 1 
\end{cases} \)

(a) Find (with explanation) the numbers at which \( f \) is discontinuous.

Since \( 2-x \), \( x^2 + 2 \), and \( 3x+1 \) are each continuous for all reals, the only possible numbers where \( f \) may be discontinuous are \(-1 \) or \(+1 \).

\( a = -1 \):
(i) \( f(-1) = (-1)^2 + 2 = 3 \) (defined)
(ii) \( \lim_{x \to -1^+} f(x) = \lim_{x \to -1} x^2 + 2 = 3 \) (singed)
(iii) \( \lim_{x \to -1^-} f(x) = \lim_{x \to -1} 2-x = 3 \) (singed)
\( a = 1 \):
(i) \( f(1) = 3(1) + 1 = 4 \) (defined)
(ii) \( \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 + 2 = 3 \) (singed)
(iii) \( \lim_{x \to 1^-} f(x) = \lim_{x \to 1} 2-x = 3 \) (singed)
\( a = -1 \) so \( f \) is continuous at \( a = -1 \).

(b) At which of these points (that is, the numbers found in part (a)) is \( f \) continuous from the right, from the left, or neither?

Since \( f(1) = 4 = \lim_{x \to 1^+} f(x) \), we see \( f \) is continuous from the right at \( a = 1 \).

(c) Sketch the graph of \( f \).
6. (6 pts.) Use the Intermediate Value Theorem to show that there is a root of the equation \(3x^4 - 5x^2 + x - 1 = 0\) in the interval \((-1, 3)\).

Let \(f(x) = 3x^4 - 5x^2 + x - 1\). Then \(f\) is continuous on \([-1, 3]\). Also, 
\[f(-1) = 3(-1)^4 - 5(-1)^2 + (-1) - 1 = -3\] and 
\[f(3) = 3(3)^4 - 5(3)^2 + 3 - 1 = 200\]. For \(N = 0\), which is between \(-3\) and \(200\), the Intermediate Value Theorem guarantees a number \(c\) in the interval \((-1, 3)\) such that \(f(c) = 0\). Thus, \(3c^4 - 5c^2 + c - 1 = 0\).

7. (8 pts.) Calculate the appropriate limits to find the horizontal and vertical asymptotes of the curve \(y = \frac{3x-1}{2-5x}\). Then graph the curve using this information.

\[
\begin{align*}
\text{Vertical asymptotes:} & \quad x = \frac{2}{5} \\
\text{Horizontal asymptote:} & \quad y = \frac{3}{5} \\
\text{Horizontal asymptote:} & \quad y = \frac{3}{5} \\
\text{Asymptote:} & \quad y = \frac{3}{5}
\end{align*}
\]

8. (10 pts.) Find an equation of the tangent line to the graph of \(y = \sqrt{1 - 2x}\) at the point \((-1, \sqrt{3})\).

\[
\begin{align*}
M_{tan} &= \lim_{h \to 0} \frac{f(-1 + h) - f(-1)}{h} \\
&= \lim_{h \to 0} \frac{\sqrt{1 - 2(-1 + h)} - \sqrt{3}}{h} \\
&= \lim_{h \to 0} \frac{\sqrt{1 + 2 - 2h} - \sqrt{3}}{h} \\
&= \lim_{h \to 0} \frac{\sqrt{3 - 2h} - \sqrt{3}}{h} \cdot \frac{(\sqrt{3 - 2h} + \sqrt{3})}{(\sqrt{3 - 2h} + \sqrt{3})} \\
&= \lim_{h \to 0} \frac{3 - 2h - 3}{h(\sqrt{3 - 2h} + \sqrt{3})} \\
&= \lim_{h \to 0} \frac{-2}{\sqrt{3 - 2h} + \sqrt{3}} \\
&= \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}}
\end{align*}
\]
9. (8 pts.) A particle moves along a straight line with equation of motion \( s = f(t) = 3t^2 - 4t + 2 \), where \( s \) is measured in meters and \( t \) in seconds. Find the velocity when \( t = 2 \) seconds. 

\[
\begin{align*}
V(t) &= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h \to 0} \frac{3(2+h)^2 - 4(2+h) + 2 - 6}{h} \\
&= \lim_{h \to 0} \frac{3(4+4h+h^2) - 8 - 4h - 4}{h} \\
&= \lim_{h \to 0} \frac{12 + 12h + 3h^2 - 12 - 4h}{h} \\
&= \lim_{h \to 0} \frac{8h + 3h^2}{h} = \lim_{h \to 0} \frac{h(8+3h)}{h} = \lim_{h \to 0} 8 + 3h = 8
\end{align*}
\]

10. (10 pts.) Find the derivative of \( f(x) = \frac{x+2}{x-3} \) using the definition of derivative. State the domain of the function and the domain of its derivative.

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\frac{x+h+2}{x+h-3} - \frac{x+2}{x-3}}{h} \\
&= \lim_{h \to 0} \frac{(x+h+2)(x-3) - (x+2)(x+h-3)}{h(x+h-3)(x-3)} \\
&= \lim_{h \to 0} \frac{x^2 - 3x + x^2 + 2x + 2x - 3 - x^2 - 3x - 2x + 6}{h(x+h-3)(x-3)} \\
&= \lim_{h \to 0} \frac{-5h}{h(x+h-3)(x-3)} = \lim_{h \to 0} \frac{-5}{(x+h-3)(x-3)} = \frac{-5}{(x-3)^2}
\end{align*}
\]

\( f'(x) = \frac{-5}{(x-3)^2} \) 

Domain: \((-\infty, 3) \cup (3, \infty)\)