4.1 Exponential functions

An exponential function has the form \( y = a^x \) where \( a \neq 0 \) and \( a \neq 1 \).

Recall:

If \( x = n \) is a positive integer, then \( a^x = \)

If \( x = 0 \), then \( a^0 = \)

If \( x = -n \), where \( n \) is a positive integer, then \( a^x = \)

If \( x = \frac{p}{q} \) is rational (\( p \) and \( q \) are integers, \( q > 0 \)), then \( a^x = \)

Now: What if \( x \) is irrational? (\( x \) is a real number, but not rational)

\( 2^x = ? \)

**EXAMPLES** Graph the function.

1. \( f(x) = 2^x \)

2. \( g(x) = \left( \frac{1}{2} \right)^x \)

Graphs of any exponential function:
EXAMPLES  Use transformations to graph each of the following functions.

1. \( f(x) = 2^{x+5} \)

2. \( g(x) = 3^{-x} - 1 \)

3. \( h(x) = -\left(\frac{1}{2}\right)^x + 4 \)

EXAMPLE  A certain breed of mouse was introduced onto a small island with an initial population of 320 mice, and scientists estimate that the mouse population is doubling every year.

(a) Find a function that models the number of mice after \( t \) years.

(b) Estimate the mouse population after 8 years.

Compound interest: \( A(t) = P\left(1+\frac{r}{n}\right)^{nt} \)

- \( A(t) \) is amount after \( t \) years
- \( P \) is principal
- \( r \) is interest rate per year
- \( n \) is number of times interest is compounded per year
- \( t \) is number of years

EXAMPLE  If $1000 is invested at an interest of 1.25% per year, compounded quarterly, find the value of the investment after the given number of years.

(a) 1 year  (b) 2 years  (c) 10 years