1.5 Continuity

Continuous function – “can draw the entire graph without having to lift your pencil off the paper”

Examples of discontinuous functions:

Continuity at a point $x = a$: A function $f$ is **continuous at** $x = a$ if

(i)

(ii)

and (iii)

If $f$ is continuous at every point in an interval $(a, b)$, then $f$ is **continuous on** $(a, b)$.

A function $f$ is **continuous from the right** if

(i)

(ii)

and (iii)

A function $f$ is **continuous from the left** if

(i)

(ii)

and (iii)

**EXAMPLES**

1. Is $f(x) = \begin{cases} x^2 & x \leq 2 \\ x + 2 & x > 2 \end{cases}$ continuous at $x = 2$?

2. Is $g(x) = \begin{cases} 3x^2 + 2 & x < 1 \\ x - 4 & x \geq 1 \end{cases}$ continuous at $x = 1$?
FACT  If $f$ is continuous on $(a, b)$ and $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$, then $f$ is continuous on $[a, b]$.

THEOREM  If $f$ and $g$ are continuous at $a$, then so are $f + g$, $f - g$, $fg$, $\frac{f}{g}$ ($g(a) \neq 0$).

Continuous functions  (continuous on their domains)
   polynomials, rational, trig, inverse trig, exponential, logarithms

THEOREM  If $g$ is continuous at $a$ and $f$ is continuous at $g(a)$, then $(f \circ g)$ is continuous at $a$.

EXAMPLES  Where (on what intervals) are the following functions continuous?
1. $f(x) = \frac{x^2 - 4}{x + 2}$
2. $g(x) = 2^{3x} \cos x$

EXAMPLE  Find $\lim_{x \to 4} \frac{5 + \sqrt{x}}{\sqrt{5 + x}}$

Intermediate Value Theorem  Let $f$ be continuous on $[a, b]$. Suppose that $f(a) \neq f(b)$. If $N$ is any number between $f(a)$ and $f(b)$, then there exists a number $c$ between $a$ and $b$ such that $f(c) = N$.

EXAMPLE  Consider $f(x) = x^2 - 2x$ on $[1, 5]$. Illustrate the Intermediate Value Theorem.