Chapter 6

Interest Rate Parity

PROBLEMS

1. In the entry forms for its contests, Publisher’s Clearing House states, “You may have already won $10,000,000.” If the Prize Patrol visits your house to inform you that you have won, it offers you $333,333.33 each and every year for 30 years. If the interest rate is 8% p.a., what is the actual present value of the $10,000,000 prize?

   Answer: The present value of 30 annual payments of $333,333.33 when discounted at 8% is
   \[
   \sum_{k=1}^{30} \frac{333,333.33}{1.08^k} = 3,752,594.41
   \]

   This value can be found in Excel by using the function NPV(rate, cashflows), where rate = 8% and cashflows refers to a sequence of 30 cells that all have the value $333,333.33.

2. Suppose the 5-year interest rate on a dollar-denominated pure discount bond is 4.5% p.a., whereas in France, the euro interest rate is 7.5% p.a. on a similar pure discount bond denominated in euros. If the current spot rate is $1.08/€, what is the value of the forward exchange rate that prevents covered interest arbitrage?

   Answer: We know that the 5-year forward rate must satisfy
   \[
   F(t,5,$/€) = S(t,$/€) \times \frac{(1+i(t,5,$))^{5}}{(1+i(t,5,€))^{5}} = 1.08/€ \times \frac{1.045^5}{1.075^5} = 0.9375/€
   \]

3. Carla Heinz is a portfolio manager for Deutsche Bank. She is considering two alternative investments of EUR10,000,000: 180-day euro deposits or 180-day Swiss francs (CHF) deposits. She has decided not to bear transaction foreign exchange risk. Suppose she has the following data: 180-day CHF interest rate, 8% p.a., 180-day EUR interest rate, 10% p.a., spot rate EUR1.1960/CHF, 180-day forward rate, EUR1.2024/CHF. Which of these deposits provides the higher euro return in 180 days? If these were actually market prices, what would you expect to happen?

   Answer: The euro return to investing directly in euros is 5% = \( \left( 10\% \times \frac{180}{360} \right) \), so the euros available in 180 days is EUR10,000,000 \( \times \) 1.05 = EUR10,500,000. Alternatively, the
EUR10,000,000 can be converted into Swiss francs at the spot rate of EUR1.1960/CHF. The Swiss francs purchased would equal EUR10,000,000 / EUR1.1960/CHF = CHF8,361,204. This amount of Swiss francs can be invested to provide a $4\% = \left( \frac{8\% \times 180}{360} \right)$ return over the next 180 days. Hence, interest plus principal on the Swiss francs is CHF8,361,204 $\times$ 1.04 = CHF8,695,652. If we sell this amount of Swiss francs forward for euros at the 180-day forward rate of EUR1.2024/CHF, we get a euro return of CHF8,695,652 $\times$ EUR1.2024/CHF = EUR10,455,652. This is less than the return from investing directly in euros.

If these were the actual market prices, you should expect investors to do covered interest arbitrages. Investors would borrow Swiss francs, which would tend to drive the CHF interest rate up; they would sell the Swiss francs for euros in the spot foreign exchange market, which would tend to lower the spot rate of EUR/CHF; they would deposit euros, which would tend to drive the EUR interest rate down; and they would contract to buy CHF with EUR in the 180-day forward market, which would put upward pressure on the forward rate of EUR/CHF. Each of these actions would help bring the market back to equilibrium.

4. If the 30-day yen interest rate is 3% p.a., and the 30-day euro interest rate is 5% p.a., is there a forward premium or discount on the euro in terms of the yen? What is the magnitude of the forward premium or discount?

Answer: We know that the high interest rate currency must sell at a forward discount when priced in the low interest rate currency to prevent a covered interest arbitrage. Therefore the euro is at a discount in the forward market. To determine the magnitude of the discount, recognize that interest rate parity requires equality of the return to investing in yen versus converting the yen principal into euros, investing the euros, and selling the euro principal plus interest in the forward market for yen:

\[
(1 + i(¥)) = \frac{1}{S(¥/€)} \times (1 + i(€)) \times F(¥/€)
\]

Solving this expression for the forward premium, we find

\[
\frac{F(¥/€) - S(¥/€)}{S(¥/€)} = \frac{i(¥) - i(€)}{(1 + i(€))}
\]

The de-annualized interest rates are 0.0025 = (3/100) $\times$ (30/360) for the yen and 0.004167 = (5/100) $\times$ (30/360) for the euro. The right-hand side of the above expression is therefore -0.00166. The annualized value is -0.00166 $\times$ (100) $\times$ (360/30) = -1.99%. We therefore say that the euro sells at an annualized discount of 1.99%.

5. Suppose the spot rate is CHF1.4706/$ in the spot market, and the 180-day forward rate is CHF1.4295/$. If the 180-day dollar interest rate is 7% p.a., what is the annualized 180-day interest rate on Swiss francs that would prevent arbitrage?
Answer: Interest rate parity requires equality of the return to investing in CHF versus converting the CHF principal into dollars, investing the dollars, and selling the dollar principal plus interest in the forward market for CHF:

\[
(1 + i(\text{CHF})) = \frac{1}{S(\text{CHF}/\text{S})} \times (1 + i(\text{S})) \times F(\text{CHF}/\text{S})
\]

If we de-annualize the dollar interest rate, we find that the 180 day interest rate is 0.035. Hence, the Swiss franc interest rate that prevents arbitrage is

\[
i(\text{CHF}) = \frac{1}{\text{CHF1.4706}/\text{S}} \times 1.035 \times \text{CHF1.4295}/\text{S} \times 1 = 0.0061
\]

If we annualize this value, we find 0.0061 \times (100) \times (360/180) = 1.21%.

6. As a trader for Goldman Sachs you see the following prices from two different banks:

<table>
<thead>
<tr>
<th>1-year euro deposits/loans:</th>
<th>6.0% – 6.125% p.a.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-year Malaysian ringgit deposits/loans:</td>
<td>10.5% – 10.625% p.a.</td>
</tr>
</tbody>
</table>

The interest rates are quoted on a 360-day year. Can you do a covered interest arbitrage?

Answer: We need to check the two inequalities that characterize the absence of covered interest arbitrage. In the first, we will borrow euros at 6.125%, convert to ringgits in the spot market at MYR4.6602 / EUR, invest the ringgits at 10.5%, and sell the ringgit principal plus interest forward for euros at MYR4.9650 / EUR. We find that

\[
1.06125 \times \frac{\text{MYR4.6602}}{\text{EUR}} \times 1.105 \times \frac{1}{\text{MYR4.9650/EUR}} = 1.0372
\]

Thus, it is not profitable to try to arbitrage in this direction as the amount that we would owe is greater than the amount that we would gain.

Let’s try the other direction, arbitraging out of ringgits into euros and covering the foreign exchange risk. We will borrow ringgits at 10.625%, convert to euros in the spot market at MYR4.6622 / EUR, invest the euros at 6.0%, and sell the euro principal plus interest forward for ringgits at MYR4.9500 / EUR. We find that

\[
1.10625 < \frac{1}{\text{MYR4.6622/EUR}} \times 1.06 \times \frac{\text{MYR4.9500}}{\text{EUR}} = 1.1254
\]

Thus, there is a possible arbitrage opportunity because the amount that we owe from borrowing ringgits is less than the amount that we gain by converting from ringgits to euros, investing the euros, and covering the transaction exchange risk with a forward sale of euros for ringgits.

©2012 Pearson Education, Inc.
7. As an importer of grain into Japan from the United States, you have agreed to pay $377,287 in 90 days after you receive your grain. You face the following exchange rates and interest rates: spot rate, ¥106.35/$, 90-day forward rate ¥106.02/$, 90-day USD interest rate, 3.25% p.a., 90-day JPY interest rate, 1.9375% p.a.

a. Describe the nature and extent of your transaction foreign exchange risk.

*Answer:* As a Japanese grain importer, you are contractually obligated to pay $377,287 in 90 days. Any weakening of the yen versus the dollar will increase the yen cost of your grain. The possible loss is unbounded.

b. Explain two ways to hedge the risk.

*Answer:* You could hedge your risk by buying dollars forward at ¥106.02/$. Alternatively, you could determine the present value of the dollars that you owe and buy that amount of dollars today in the spot market. You could borrow that amount of yen to avoid having to pay today.

c. Which of the alternatives in part b is superior?

*Answer:* If you do the forward hedge, you will have to pay
\[ ¥106.02/\$ \times 377,287 = ¥39,999,967.74 \]
in 90 days. If you do the money market hedge, you first need to find the present value of $377,287 at 3.25%. The de-annualized interest rate is \((3.25/100) \times (90/360) = 0.008125\). Thus, the present value is
\[ \frac{377,287}{1.008125} = 374,246.25 \]
Purchasing this amount of dollars in the spot market costs
\[ ¥106.35/\$ \times 374,246.25 = ¥39,801,088.69 \]
To compare this value to the forward hedge, we must take its future value at 1.9375% p.a. The de-annualized interest rate is \((1.9375/100) \times (90/360) = 0.00484375\), and the future value is
\[ ¥39,801,088.69 \times (1.00484375) = ¥39,993,875.21 \]
The cost of the money market hedge is essentially the same as the cost of the forward hedge because interest rate parity is satisfied.

8. You are a sales manager for Motorola and export cellular phones from the United States to other countries. You have just signed a deal to ship phones to a British distributor. The deal is denominated in pounds, and you will receive £700,000 when the phones arrive in London in 180 days. Assume that you can borrow and lend at 7% p.a.
in U.S. dollars and at 10% p.a. in British pounds. Both interest rate quotes are for a 360-day year. The spot exchange rate is $1.4945/£, and the 180-day forward exchange rate is $1.4802/£.

a. Describe the nature and extent of your transaction foreign exchange risk.

Answer: As a U.S. exporter, you have a contract to receive £700,000 in 180 days. Any weakening of the pound versus the dollar will decrease the dollar value of your pound-denominated receivable. Large losses are possible as the dollar value could go to zero, although that is highly unlikely.

b. Describe two ways of eliminating the transaction foreign exchange risk.

Answer: You could hedge by selling pounds forward for dollars. Alternatively, you could do a money market hedge in which you borrow the present value of the pounds, and convert the loan principal to dollars in the spot market, and then use the pound receivable to pay off the interest plus principal on the loan at maturity.

c. Which of the alternatives in part b is superior?

Answer: The forward hedge gives

\[ $1.4802/£ \times £700,000 = $1,036,140 \]

in 180 days. The money market hedge requires the present value of the £700,000. The interest rate is \( (10/100) \times (180/365) = 0.0493 \). Thus, the present value is

\[ £700,000 / 1.0493 = £667,111.41 \]

The dollar value of this is

\[ $1.4945/£ \times £667,111.41 = $996,998 \]

To compare this to the forward hedge we must take its future value at 7% p.a. The interest rate is \( (7/100) \times (180/360) = 0.035 \). Therefore the future value is

\[ $996,998 \times 1.035 = $1,031,892.93 \]

The forward hedge provides slightly more dollar revenue.

d. Assume that the dollar interest rate and the exchange rates are correct. Determine what sterling interest rate would make your firm indifferent between the two alternative hedges.

Answer: We know that if interest rate parity is satisfied, the money market hedge and the forward hedge will provide the same revenue. The pound interest rate that satisfies interest rate parity is
\[
(1 + i(\£)) = S(\$/\£) \times (1 + i(\$)) \times \frac{1}{F(\$/\£)}
\]

The value of the right-hand side is \$1.4945/\£ \times 1.035 / \$1.4802/\£ = 1.0450. Thus the annualized pound interest rate that would make the firm indifferent between the forward hedge and the money market hedge is \(0.0450 \times \frac{100}{365/180} = 9.12\%\).

9. Suppose that there is a 0.5% probability that the government of Argentina will nationalize its banking system and freeze all foreign deposits indefinitely during the next year. If the dollar deposit interest rate in the United States is 5%, what dollar interest would Argentine banks have to offer in order to attract deposits from foreign investors?

\textit{Answer:} If the freezing of deposits is an idiosyncratic event, then the expected value of the return should equal the risk free return of 5%. If investors effectively get a return of zero with 0.5% probability, they must get a return of \((1 + X\%)\) with 99.5% probability, such that

\[
[(1 + X\%) \times 0.995] + [0 \times 0.005] = 1.05
\]

When we solve this equation for \(X\%), we find \(X\% = 5.53\%\). Of course, the more that you eventually recover in the event of a freeze of deposits, the smaller the interest rate can be.

10. Suppose the market price of a 20-year pure discount bond with a face value of $1,000 is $214.55. What is the spot interest rate for the 20-year maturity expressed in percentage per annum?

\textit{Answer:} We know that the relationship between the price of a pure discount bond and the spot interest rate at the 20 year maturity satisfies

\[
P(t) = \frac{\$1,000}{(1 + i(t,20))^{20}}
\]

Substituting the price of $214.55 and solving for \(i(t,20)\), we find

\[
i(t,20) = \left[ \frac{\$1,000}{\$214.55} \right]^{1/20} - 1 = 0.08
\]

Therefore, the spot interest rate for the 20-year maturity expressed in percentage per annum is 8%.

11. Consider a 2-year euro-denominated bond that has a current market price of €970, a face value of €1,000, and an annual coupon of 5%. Suppose the 1-year euro-denominated spot interest rate is 5.5%. What is the 2-year euro-denominated spot interest rate?
Answer: The present value of a coupon paying bond is found by discounting each annual coupon and the final principal payment at the appropriate spot interest rates for those maturities. Thus, to find the 2-year euro-denominated spot interest rate we must solve for the two-period spot interest rate in the following equation:

$$\frac{\text{€970}}{1.055} + \frac{\text{€1050}}{(1+i(t,2))^2}$$

The answer is $i(t,2) = 6.68\%$. 