PROBLEMS

1. The EAFE is the international index comprising markets in Europe, Australia, and the Far East. Consider the following annualized stock return data:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Average U.S. index return:</td>
<td>14%</td>
</tr>
<tr>
<td>Average EAFE index return:</td>
<td>13%</td>
</tr>
<tr>
<td>Volatility of the U.S. return:</td>
<td>15.5%</td>
</tr>
<tr>
<td>Volatility of the EAFE return:</td>
<td>16.5%</td>
</tr>
<tr>
<td>Correlation of U.S return and EAFE return:</td>
<td>0.45</td>
</tr>
</tbody>
</table>

a. What would be the return and risk of a portfolio invested half in the EAFE and half in the U.S. market?

*Answer:* Using standard formulas for the expected return and volatility of a portfolio of two assets, we find:

\[ E[r^p] = (0.5)E[r_{U.S.}] + (0.5)E[r_{EAFE}] = (0.5)(14\%) + (0.5)(13\%) = 13.5\% \]

\[
\text{VOL}[r^p] = \sqrt{(0.5)^2(15.5\%)^2 + (0.5)^2(16.5\%)^2 + 2(0.5)^2(0.45)(15.5\%)(16.5\%)}
\]

\[
= 0.5\sqrt{(15.5\%)^2 + (16.5\%)^2 + 2 \times 0.45 \times 15.5\% \times 16.5\%}
\]

\[ = 13.63\% \]

Note that this is lower than the volatility of either of the two indexes.

b. Market watchers have noticed slowly increasing correlations between the United States and the EAFE index, which some ascribe to the increasing integration of markets. Given that the volatilities remain unchanged, is it possible that the volatility of a portfolio that is equally weighted between the two indexes has higher volatility than the U.S. market?

*Answer:* Yes. For example, when \( \rho = 1.00 \), the variability of the equally weighted portfolio would just be the average volatility. There would be no risk reduction through diversification.

4. Thom Yorke is a typical mean-variance investor, currently invested 100% in a diversified U.S. equity portfolio with expected return of 12.46% and volatility of
15.76%. Thom is considering adding the STCMM fund to his portfolio. STCMM invests in U.S. small-capitalization, high technology firms and has an expected of 14.69% and a volatility of 32.5%. Thom has determined its correlation with his current portfolio to be 0.7274. He is also intrigued by the LYMF fund, which invests in several emerging markets. The expected return on the fund is only 12%; it has 35% volatility and a correlation of 0.2 with his portfolio. The correlation of the LYMF fund with the STCMM fund is 0.15. Assume that the risk-free rate is 5%.

a. If Thom is interested in improving the Sharpe ratio of his portfolio, will he invest a positive amount in one of the funds? Which one? Carefully explain your reasoning.

Answer: We established that you will add an asset to your portfolio if
\[
\frac{E[r^*] - r_f}{\text{Vol}[r^*]} > \text{corr}[r, r^*] \frac{E[r] - r_f}{\text{Vol}[r]}
\]
with * indicating the new funds.

Plugging in the numbers, we obtain:

<table>
<thead>
<tr>
<th>Investment</th>
<th>Sharpe Ratios</th>
<th>Threshold (Hurdle Rate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. portfolio</td>
<td>0.4734</td>
<td></td>
</tr>
<tr>
<td>STCMM fund</td>
<td>0.2982</td>
<td>0.3444</td>
</tr>
<tr>
<td>LYMF</td>
<td>0.2</td>
<td>0.0947</td>
</tr>
</tbody>
</table>

For example, 0.2982 = \([0.1469 - 0.05]/0.325\) and 0.3444 = 0.4734 x 0.7274.

Although the LYMF (“Lose Your Money Fast”) fund has a much lower Sharpe ratio than the STCMM (“Short-Term Money Mis-Management”) fund, it will get added to the portfolio, because of its low correlation with Thom's portfolio. The STCMM fund pretty much looks like a levered version of the portfolio Thom already has and seems not to eliminate much systematic risk.

b. Suppose Thom is more risk averse than his friend, Nick Cave. Both cannot short-sell securities, and both are thinking of splitting their entire portfolio between the U.S. portfolio that Thom is currently holding, the STCMM fund, and the LYMF fund. They also do not invest in the risk-free asset and do not consider levering up risky portfolios. Compare the two investors’ optimal holdings. Who will invest more in the LYMF fund, and who will invest more in the STCMM fund? Why?

Answer: While it is impossible to answer this question precisely without more information about how the two funds correlate, some outcomes are very likely. Maybe somewhat surprisingly the “risky” emerging markets fund will be held by the least risk-tolerant investors. Although very risky by itself, when added in small proportions to the portfolio Thom already has, the emerging market fund is a wonderful diversifier. He will surely hold the fund, since he can lower his risk without lowering expected returns much. Nick Cave on the other hand may actually invest a bit in the STCMM fund. If his preferences are such that he requires more
than 12.46% return, he must hold some of the fund, since he cannot reach a better return with the two other investments. Needless to say, he will move along the mean-standard deviation frontier in riskier territory.