MORAL HAZARD AND BACKGROUND RISK
IN COMPETITIVE INSURANCE MARKETS:
THE DISCRETE EFFORT CASE

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ABSTRACT

We examine the effect of background risk on competitive insurance markets with moral hazard. If policyholders have nonnegative prudence, then background risk does not decrease effort and, when effort increases, expands the set of feasible policies. However, the effect of background risk on equilibrium is indeterminate.

Keywords: asymmetric information, mutual insurance, randomized contracts.

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1. Introduction

It has long been known that the effects of risks are not additive and that the interactions among risks are important (e.g., Markowitz, 1959). More recently, the effects of exogenous uninsurable background risk on optimal hedging, portfolio and insurance decisions have been extensively investigated. Intuitively, we expect that the addition of, or an increase in, background risk will lead individuals to reduce their exposure to insurable risks. A great deal of attention has been paid to determining the conditions under which this intuition is correct. Gollier and Pratt (1996) provide a seminar paper on the impact of background risk on individual preferences under uncertainty; Eeckhoudt and Gollier (2000) and Gollier (2001) summarize much of the recent research. The literature has consistently assumed that the underlying distributions of the risks are given and unaffected by the decision-maker, that is, there is no moral hazard.

The objective of this paper is to examine the effect of background risk on competitive insurance markets with moral hazard. Moral hazard is a long-time concern in the insurance industry and a long-time concern to insurance economists (Arrow, 1963, Pauly, 1968). Moral hazard is the tendency of insurance coverage to reduce the individual’s incentive to prevent the occurrence of the event insured against when the loss prevention efforts cannot be observed. A partial solution to the problem of moral hazard is for insurers to offer incomplete coverage; the incomplete coverage provides an incentive for individuals to undertake some loss prevention efforts (Shavell, 1979). The
degree of incomplete coverage and the cost of that coverage depend on the effectiveness of loss prevention efforts, the cost of those efforts and attitudes toward risk.

Background risk affects the design of insurance policies through its effect on attitudes toward risk. We show that, if individuals have nonnegative prudence, then background risk increases the difference in utility between the loss and no loss states of the world. This implies that, for any insurance policy, individuals will not choose lower effort in the presence of background risk. When in fact individuals choose higher effort, background risk expands the set of policies that insurers can offer that earn non-negative expected profit. To put this differently, for any given level of insurance coverage, the premium is lower when background risk is present. However, the effect of background risk on the equilibrium policies is indeterminate. Compared to the equilibrium policy in the absence of background risk, the equilibrium policy in the presence of background risk may have more or less coverage and a higher or lower premium. As Ligon and Thistle (2006) show, all of these results hold in the continuous effort case as well. Ligon and Thistle also discuss the implication of these results for the stock-mutual policy choice and for indexed insurance products.

In the next section, we present the model allowing agents to choose their level of effort from a set of discrete effort levels. Section 3 provides brief concluding remarks.

2. Moral Hazard and Background Risk with Discrete Effort Levels

A. Two Effort Levels. In this section we begin with a simple model of insurance under moral hazard in which the policyholder can choose between high and low effort.

The model is essentially the same as in Arnott (1992) and Arnott and Stiglitz (1988a, 1991), and we make extensive use of their results. There are two possible states
of the world, loss (L) and no loss (N). Consumers have fixed initial wealth \( w_0 \) and may suffer a loss of \( l < w_0 \). We assume the amount of the loss is fixed, so that moral hazard affects only the probability of loss. Consumers can choose either low effort, \( e_0 \), or high effort \( e_1 \), where \( e_0 < e_1 \). The corresponding loss probabilities are \( p_0 \) and \( p_1 \), where \( 0 < p_1 < p_0 < 1 \). Consumers are assumed to maximize the expected value of \( u(w) - e \), an event independent, additively separable von Neumann-Morgenstern utility function over wealth and effort. We assume that individuals are nonsatiated \( (u' > 0) \) and risk averse \( (u'' < 0) \) and have non-negative prudence \( (-u'''/u'' > 0) \). The Arrow-Pratt risk premium, \( \pi(w) \), is defined by \( E\{u(w + z)\} = u(w - \pi(w)). \)\(^1\) Note that nonnegative prudence is consistent with but less restrictive than the common assumption of weakly decreasing absolute risk aversion.

An insurance policy consists of a premium \( \alpha \) paid if no loss occurs and a net indemnity \( \beta \) received in the event of a loss.\(^2\) We assume the parties are committed to the terms of the policy and are not permitted to renegotiate. Thus, a consumer with policy \((\alpha, \beta)\) who chooses effort level \( e_i \) has expected utility

\[
U(\alpha, \beta, e_i) = (1 - p_i)u(w_0 - \alpha) + p_iu(w_0 - l + \beta) - e_i.
\]

For a fixed effort level the indifference curves are upward sloping and concave in indemnity-premium space. The indifference curves are illustrated in Figure 1, where the low effort indifference curves is LL’ and the high effort indifference curve is HH’. Since the probability of loss is higher, the low effort indifference curves are steeper and more concave than the high effort indifference curves.

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\(^1\) We have suppressed the dependence of the risk premium on the risk, \( z \), for notational convenience.

\(^2\) We consider only deterministic policies. Ligon and Thistle (2006) also discuss randomized policies. We assume throughout that all claims are valid and paid in full. For an analysis of insurer non-performance (albeit without moral hazard), see Doherty and Schlesinger (1990) and Mahul and Wright (2004).
The choice between high and low effort levels depends on the terms of the insurance policy. The difference in expected utility for high and low effort is
\[
S(\alpha, \beta) = U(\alpha, \beta, e_1) - U(\alpha, \beta, e_0) = (p_0 - p_1)[u(w_0 - \alpha) - u(w_0 - l + \beta)] - (e_1 - e_0).
\]
The individual chooses high effort if \( S \geq 0 \) and chooses low effort if \( S < 0 \). The set of policies for which \( S(\alpha, \beta) = 0 \) is the “switching locus;” along the switching locus the individual is indifferent between high and low effort. The switching locus is downward sloping and may be either concave or convex. The switching locus is shown in Figure 1 as the curve SS. The individual chooses high effort to the left of the switching locus and low effort to the right of the switching locus. Since the level of effort changes with the terms of the insurance policy, the variable effort indifference curve is the curve HAL’ in Figure 1. The variable effort indifference curve cannot be globally concave.

Now consider the effect of background risk. The individual’s wealth is now \( w_0 + z \), where \( z \) is the zero mean independent background risk. The individuals’ expected utility with policy \((\alpha, \beta)\) and effort level \( e_i \) becomes
\[
U^*(\alpha, \beta, e_i) = (1 - p_i)E\{u(w_0 + z - \alpha)\} + p_iE\{u(w_0 + z - l + \beta)\} - e_i,
\]
where the expectation is with respect to \( z \). Let \( w_N = w_0 - \alpha \) and \( w_L = w_0 - l + \beta \) and let \( \pi_N = \pi(w_N) \) and \( \pi_L = \pi(w_L) \). Then we can write expected utility as
\[
U^*(\alpha, \beta, e_i) = (1 - p_i)u(w_N - \pi_N) + p_iu(w_L - \pi_L) - e_i.
\]

The effect of background risk on the choice of effort depends on how the background risk affects the switching locus. With background risk, the difference in expected utility for high and low effort levels is
\[
S^*(\alpha, \beta) = U^*(\alpha, \beta, e_1) - U^*(\alpha, \beta, e_0)
\]
\[(p_0 - p_1)[u(w_N - \pi_N) - u(w_L - \pi_L)] - (e_1 - e_0).\]

To determine the effect of background risk on effort, we need to compare \(S^*(\alpha, \beta)\) and \(S(\alpha, \beta)\). Since we use this result below, we state it formally:

**Lemma:** Nonnegative prudence implies that \([u(w_N - \pi_N) - u(w_L - \pi_L)] \geq [u(w_N) - u(w_L)]\).

**Proof:** Let function \(v\) be defined as \(v(w) = Eu(w+z) = u(w-\pi(w))\) for all \(w\). Nonnegative prudence means that \(u'\) is (at least weakly) convex, so that \(v'(w) = Eu'(w+z) \geq u'(w)\) for all \(w\), by Jensen’s inequality. This implies that

\[
\frac{u(w_N - \pi_N) - u(w_L - \pi_L)}{u(w_N) - u(w_L)} = \int_{w_L}^{w_N} v'(w)dw \\
\geq \int_{w_L}^{w_N} u'(w)dw \\
= u(w_N) - u(w_L) \\
\]

The Lemma establishes \(S^*(\alpha, \beta) \geq S(\alpha, \beta)\). When the inequality is strict, the Lemma implies that, for any policy, the introduction of background risk increases the gain from choosing high effort. In terms of Figure 1, the presence of background risk shifts the switching locus to \(S^*S^*\), which lies to the right of \(SS\).

**Proposition 1:** If the policyholder has non-negative prudence, the presence of background risk does not reduce effort.

**Proof:** In order for the presence of background risk to reduce effort, there must be policies for which \(S(\alpha, \beta) \geq 0\) and \(S^*(\alpha, \beta) < 0\). Since non-negative prudence implies \(S^*(\alpha, \beta) \geq S(\alpha, \beta)\), this is not possible.
The intuition for this result is straightforward. The presence of background risk increases the utility gain of being in the no loss state, strengthening the incentive to choose the high effort level. If the individual chooses high effort in the absence of background risk ($S \geq 0$), then they choose high effort in the presence of background risk ($S^* \geq 0$). There may exist policies for which the individual chooses low effort in the absence of background risk but high effort in the presence of background risk. Finally, there are policies (e.g., full coverage policies) for which the individual chooses low effort in both the absence and the presence of background risk.

Insurance companies are assumed to be risk neutral expected profit maximizers. Background risk affects the policies that insurance companies can profitably offer, that is, the set of feasible policies. The insurance company needs to take account of the effect of the terms of the policy on policyholders’ effort and loss probabilities. The problem is easily illustrated using Figure 2. The line $0ABC$ is the fixed effort zero profit line for high effort where the loss probability is $p_1$. The line $0DEF$ is the fixed effort zero profit line for low effort where the loss probability is $p_0$. The line $FC$ is the full coverage locus ($\alpha + \beta = l$). If there is no background risk, then the switching locus is $SS$. The set of feasible policies, given that policyholders change their effort level, is then the policies on or above $0ADF$. Observe that the feasible set is not convex. If there is background risk, then the switching locus is $S*S^*$. The feasible set becomes the set of policies on or above $0BEF$.

This leads to the following result.

**Proposition 2:** With non-negative prudence, the presence of background risk does not contract the set of feasible policies.
Proof: Since the presence of background risk does not reduce effort, any policy that earns non-negative expected profit in the absence of background risk earns non-negative expected profit in the presence of background risk. If the inequality of the Lemma is strict, effort increases at less than full coverage and there are policies, such as those along AB in Figure 2, that earn negative expected profit in the absence of background risk that earn non-negative expected profit in the presence of background risk, which strictly expands the set of feasible policies. ||

Background risk also affects the competitive equilibrium in the insurance market. If insurance policies are actuarially fairly priced, then, for fixed effort, expected utility increases as the level of coverage increases. This implies that, in the absence of background risk, there are two possible competitive equilibria. The first equilibrium is at the intersection of the high effort fair price line and the switching locus, point A in Figure 2. At this equilibrium, individuals receive partial coverage and choose high effort. The other equilibrium is at full coverage on the low effort fair price line, point F in Figure 2. At this equilibrium individuals receive full coverage and choose low effort.

Using the fact that individuals are indifferent between high and low effort at A, whether the equilibrium is at A or F depends on whether the expected utility of low effort is higher at A or F. That is, let $T(\alpha_A, \beta_A) = U(\alpha_A, \beta_A, e_0) - U(p_0l, (1 - p_0)l, e_0)$, where $\alpha_A = [p_1/(1 - p_1)]\beta_A$. Then the equilibrium is at A if $T \geq 0$ and at F if $T < 0$. The presence of background risk shifts the switching locus to $S^*S^*$. The two possible equilibria are with partial coverage and high effort at B and with full coverage and low effort at F. Now let $T^*(\alpha_B, \beta_B) = U^*(\alpha_B, \beta_B, e_0) - U^*(p_0l, (1 - p_0)l, e_0)$, where $\alpha_B = [p_1/(1 - p_1)]\beta_B$. The equilibrium is at B if $T^* \geq 0$ and at F if $T^* < 0$. 
The effect of background risk on the equilibrium depends on the relationship between \( T(\alpha_A, \beta_A) \) and \( T^*(\alpha_B, \beta_B) \). Let \( w_{NA} = w - \alpha_A \) and \( \pi_{NA} = \pi(w_{NA}) \), let \( w_{LA} = w - l + \beta_A \) and \( \pi_{LA} = \pi(w_{LA}) \) and let \( w_F = w - p_0l \) and \( \pi_F = \pi(w_F) \). Define \( w_{NB}, w_{LB}, \pi_{NB}, \) and \( \pi_{LB} \) similarly. Using a Taylor series we can then write \( T^*(\alpha_B, \beta_B) \) as

\[
T^*(\alpha_B, \beta_B) \approx \{(1 - p_0)u(w_{NB}) + p_0u(w_{LB}) - u(w_F)\}
+ \left[ -(1 - p_0)\pi_{NB}u'(w_{NB}) - p_0\pi_{LB}u'(w_{LB}) + \pi_Fu'(w_F) \right]
\]

The term in braces is \( T(\alpha_B, \beta_B) \) and we let \( C(\alpha_B, \beta_B) \) denote the second term. Background risk has two effects on the equilibrium, an indirect effect on the level of coverage captured by \( T \) and a direct effect captured by \( C \).

Since the indemnity is higher at B than at A and the premium is fair, we have \( T(\alpha_B, \beta_B) > T(\alpha_A, \beta_A) \). Using the fact that \( \pi(w) \approx \frac{1}{2} \sigma^2 \frac{z}{2} [-u''(w)/u'(w)] \), we can rewrite \( C \) as

\[
C(\alpha_B, \beta_B) = \frac{1}{2} \sigma^2 [-(1 - p_0)u''(w_{NB}) + p_0u''(w_{LB}) - u''(w_F)].
\]

The sign of \( C \) is indeterminate.

This leads to the following essentially negative result.

**Proposition 3:** In general, \( T(\alpha_A, \beta_A) \geq 0 \) \((< 0)\) is neither necessary nor sufficient for \( T^*(\alpha_B, \beta_B) \geq (\leq 0) \).

If the equilibrium in the absence of background risk as at the partial (full) coverage policy, the equilibrium in the presence of background risk may be at either the partial or full coverage policy. Conversely, if the equilibrium in the presence of background risk as at the partial (full) coverage policy, the equilibrium in the absence of background risk may be at either the partial or full coverage policy.
B. Multiple Discrete Effort Levels. These results extend directly to the case where the policyholder can choose among a discrete number of levels of effort. Suppose consumers can choose among effort levels $e_0, e_1, \ldots, e_n$, where $e_0 < e_1 < \ldots < e_n$. The corresponding loss probabilities are $p_0, p_1, \ldots, p_n$, where $0 < p_n < \ldots < p_1 < p_0 < 1$. Thus, in the absence of background risk, the switching locus between effort levels $e_{i-1}$ and $e_i$ is the set of policies for which

$$S_i(\alpha, \beta) = U(\alpha, \beta, e_i) - U(\alpha, \beta, e_{i-1})$$

is such that

$$S_i(\alpha, \beta) = (p_{i-1} - p_i)[u(w_N) - u(w_L)] - (e_i - e_{i-1}) = 0$$

for $i = 1, \ldots, n$. In the presence of background risk, the $i^{th}$ switching locus becomes

$$S_i^*(\alpha, \beta) = (p_{i-1} - p_i)[u(w_N - \pi_N) - u(w_L - \pi_L)] - (e_i - e_{i-1})$$

As before, the Lemma implies that $S_i^*(\alpha, \beta) \geq S_i(\alpha, \beta)$. Using the same arguments, Propositions 1, 2 and 3 still hold, that is, the presence of background risk does not reduce effort, expands the set of feasible policies when effort increases, and the equilibrium premium and indemnity may either increase or decrease.

3. Summary and Conclusion.

In this paper, we examine the effect of background risk on competitive insurance markets with moral hazard. We show that, if individuals have separable utility and non-negative prudence, then background risk does not decrease the level of effort chosen for any given insurance policy. When individuals undertake greater effort in the presence of background risk, the set of insurance policies that earn non-negative expected profit expands. When there is moral hazard, the equilibrium policy in the presence of background risk may involve either more coverage or less coverage and either a higher premium or a lower premium compared to the equilibrium policy in the absence of
background risk.

We should point out that background risk does not fundamentally change the analysis of competitive insurance markets with moral hazard. When effort is discrete, the variable effort indifference curves are not concave and the feasible set is not convex. Equilibrium may be at either partial or full coverage. Equilibrium exists but may or may not be unique.
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Figure 1. The Effect of Background Risk on the Choice of Effort
Figure 2. The Effect of Background Risk on the Feasible Set