Chapter 5

Risk Handling Techniques: Diversification and Hedging

Risk Bearing Institutions

- Bearing risk collectively
- Diversification
- Examples:
  - Pension Plans
  - Mutual Funds
  - Insurance Companies

Additional Benefits

- Professional management
- Administrative Services
- Investment in Information
- Investment in Infrastructure
State of the Economy Table

- Shows returns in different situations with associated probability
- Can calculate $E[R]$ and $\sigma$ again of investments given the different states of the economy
- Note that this is a discrete probability distribution

Creating a Portfolio

- What matters is interrelationship between investments across different states of the economy:

$$COV = \sum_{i=1}^{n} (r_{xi} - E[r_x]) \times (r_{yi} - E[r_y]) \times p_i$$

Correlation Coefficient

- A number that tells us whether two investments are statistically dependent:

$$\rho = \frac{COV_{AB}}{\sigma_A \times \sigma_B}$$
Chapter 5   Page 3

**Correlation Coefficient**

-1 ≤ ρ ≤ +1

- Implications:
  - Positive correlation
  - Negative correlation
  - Uncorrelated

**Diversification Again**

**TABLE 5-4** The Standard Deviation in Risk Pools with Independent and Positively Correlated Losses (in dollars)

<table>
<thead>
<tr>
<th>Size of Pool</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>100</th>
<th>10,000</th>
<th>Infinite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td>30,000</td>
<td>21,212.20</td>
<td>15,000</td>
<td>3,000</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
<td>Correlation = 0.1</td>
<td>30,000</td>
<td>22,248.59</td>
<td>17,102.63</td>
<td>9,904.54</td>
<td>9,491.10</td>
<td>9,486.83</td>
</tr>
</tbody>
</table>

**Lessons**

- Perfect positive correlation among risk exposures is unlikely
- Natural diversification occurs across uncorrelated risks
- Bundling negatively correlated risk exposures dramatically reduce risk, see Hedging
Hedging

- Taking two financial positions simultaneously whose gains will offset each other.
- Hence $\rho = -1$
- Examples:
  - Currency Risk
  - Interest Rate Risk
  - Commodity Price Risk

Derivatives

- A derivative is any asset whose payoff, price or value depends on the payoff, price or value of another asset
  - Futures Contract = Order to buy or sell an asset later at a specified price
  - Forward Contract = Same, but not traded on an organized exchange
  - Call Option = gives the holder the right to buy the underlying asset at a specified price at/until a specified date
  - Put Option = give the holder the right to sell the underlying asset at a specified price at/until a specified date
  - Swaps = Counterparties exchange cash flows of one party's financial instrument for those of the other party's financial instrument.

Derivatives

- Derivative is typically priced using “no arbitrage” arguments.
  - Arbitrage is a trading strategy that is self-financing (requires no cash) and which has a positive probability of positive profit and zero probability of negative profit.
  - That is, you get something for nothing.
  - The price of the derivative must be such that there are no arbitrage opportunities.
Forwards and Futures

- Increase in Payoff
- Long position in asset
- Increase in Price

Forward prices have the following relation to spot prices:

\[ F_t = S_t(1 + c)^T \]

where:
- \( F_t \) = forward price
- \( S_t \) = spot price
- \( c \) = carrying cost (per period)
- Opportunity cost and/or storage costs
- \( T \) = number of periods

This is an example of a No Arbitrage condition

How do you make money if \( * \) does not hold?
Options – Long Call

Payoff at expiration is \( \text{Max}[S - K, 0] - C \)

Options – Short Call

Payoff at expiration is \( -\text{Max}[S - K, 0] + C = \text{Min}[K - S, 0] + C \)

Options – Long Put

Payoff at expiration is \( \text{Max}[K - S, 0] - P \)
Chapter 5   Page 7

---

Options – Short Put

Payoff at expiration is \(-\text{Max}[K - S, 0] + P = \text{Min}[S - K, 0] + P\)

---

Options

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy call</td>
<td>Expect (P) to rise (it to fall)</td>
</tr>
<tr>
<td>Write put</td>
<td>Expect (P) won’t rise (it won’t fall)</td>
</tr>
<tr>
<td>Write call</td>
<td>Expect (P) won’t rise (it won’t fall)</td>
</tr>
<tr>
<td>Buy put</td>
<td>Expect (P) to fall (it to rise)</td>
</tr>
</tbody>
</table>

---

Some Corporate Finance

- Call option: gives holder right, but not obligation, to buy asset at exercise price.

---
Some Corporate Finance

- Increasing the riskiness of the underlying asset increases the value of the call option
  - downside risk cut off, capture upside gain

Some Corporate Finance

- What happens as incr. riskiness of asset?

![Graph showing relationship between V and A]

Some Corporate Finance

- Application to corp. fin.: If a firm has debt, then equity has same payoff structure as a call option
  - exercise price = face value of debt

- Equity is a call option on the firm’s assets,
  - Exercise price = face value of debt
Some Corporate Finance

Why?
- Shareholders can choose to default
  - If assets worth less than debt, then default
    - Let debtholders keep the assets
  - If assets worth more than debt, then pay debt
    - "Buy" assets from debtholders for D.

Some Corporate Finance

- Increasing riskiness of assets increases value of option (equity)
  - Increase in value is at expense of debtholders
  - $A = L + E$
- Shareholders have incentive to extract value

Some Corporate Finance

- What does this have to do with insurance?
- Insurance policies are debt-like
  - Promise to pay in the future
  - Default risk
- Stock co.: shareholders ≠ policyholders
- Mutual: shareholder = policyholders
Black-Scholes Formula

Black-Scholes Formula:

\[ C = S \text{N}(d_1) - K e^{-rT} \text{N}(d_2) \]

where

\[ d_1 = \frac{\ln(S/K) - T(r + \sigma^2/2)}{\sigma \sqrt{T}} \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]

The first term, \( S \text{N}(d_1) \), is the stock price times the hedge ratio.

The second term \( K e^{-rT} \text{N}(d_2) \) is really the present value of a loan.

The hedge ratio, \( \text{N}(d_1) \) is rather complicated to calculate since the stock price is changing constantly over time.

The expression \( \text{N}(.) \) is the cumulative normal distribution, reflecting that final value will follow some distribution.

\[ \text{N}(d_2) \] is the probability the call will finish “in the money”

\[ 1 - \text{N}(d_2) \] is the prob the call will finish “out of the money”

Will not be exercised

Apply to corporations with debt

- Equity is a call option on the firm’s assets
- Can use modified version of \( 1 - \text{N}(d_2) \) to estimate bankruptcy risk
Finally

- Risk Management at the Boardroom Level increases Consistency and Negotiating Power
- Remember: ERM is a holistic approach to risk management. Pure Risks and Speculative Risks for the firm!

<table>
<thead>
<tr>
<th>TABLE 5-1 Calculating the Covariance Using Data from a Probability Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State of Economy</strong></td>
</tr>
<tr>
<td>Growth</td>
</tr>
<tr>
<td>Recession</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 5-2 Negatively Correlated Stock Returns for Stocks C and D</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State of Economy</strong></td>
</tr>
<tr>
<td>Growth</td>
</tr>
<tr>
<td>Recession</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
</tr>
<tr>
<td>State of Economy</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Growth</td>
</tr>
<tr>
<td>Stable</td>
</tr>
<tr>
<td>Recession</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
</tr>
</tbody>
</table>