Which is better?
(1) 6% return with no risk, or
(2) 20% return with risk.

Cannot say - need to know how much risk comes with the 20% return.

What do we know so far?

We know why returns (rates) vary between different securities!

\[ r_{\text{nom}} = r^* + \text{IP} + \text{DRP} + \text{LP} + \text{MRP} \]

where \( r^* + \text{IP} = r_{RF} \)

But this does not quantify risk

We want to quantify risk.

Today’s concepts

(1) quantify risk
(2) tells us how much return we need for a given level of risk.

I. Risk and Return for individual assets

Risk = \( P( \text{Actual returns} < \text{Expected returns} ) \)

ex. 2 year Treasury bond - hold until maturity, pays 6%/yr

expected return = 6% [ \( \hat{r} = 6\% \) ]

If hold until maturity

Actual return = 6% [ \( r = 6\% \) ]

Therefore \( \hat{r} = r \) or \( P[r < \hat{r}]=0\% \) no risk!
Buy a 20 year STN bond to hold for 2 years

expected return = 20% \[ \hat{r} = 20\% \]

<table>
<thead>
<tr>
<th>Probability</th>
<th>Actual Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>50%</td>
<td>0%</td>
</tr>
<tr>
<td>25%</td>
<td>20%</td>
</tr>
<tr>
<td>25%</td>
<td>60%</td>
</tr>
<tr>
<td>( \hat{r} )</td>
<td>20%</td>
</tr>
</tbody>
</table>

\[ \hat{r} = \sum p_i * r_i = \]

\[ .50(.00) + .25(.20) + .25(.60) = .20 \]

How to measure risk and return:

for individual asset calculate:

\[ \hat{r} = \text{expected rate of return} \]

\[ \sigma = \text{standard deviation which measures the dispersion around the mean (measure of risk)} \]

\[ \sigma = \left[ \sum (r_i - \hat{r})^2 p_i \right]^{1/2} \]

<table>
<thead>
<tr>
<th>Instrument</th>
<th>( \hat{r} )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-Bond</td>
<td>6%</td>
<td>0%</td>
</tr>
<tr>
<td>STN Bond</td>
<td>20%</td>
<td>24.5%</td>
</tr>
</tbody>
</table>

Stock A has the following probability distribution of expected returns:

<table>
<thead>
<tr>
<th>Probability</th>
<th>Rate of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-15%</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>0.4</td>
<td>5</td>
</tr>
<tr>
<td>0.2</td>
<td>10</td>
</tr>
<tr>
<td>0.1</td>
<td>25</td>
</tr>
</tbody>
</table>

What is Stock A's expected rate of return and standard deviation?
II. Risk and Return for portfolios

**Expected Portfolio Return**

\[
\hat{r}_p = \sum x_i \cdot \hat{r}_i
\]

\(r_i = \) proportion of portfolio invested in asset \(i\).

**Portfolio standard deviation**

\[
\sigma_p = \left[ x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2x_Ax_B \rho_{AB} \sigma_A \sigma_B \right]^{1/2}
\]

where, \(x_A + x_B = 100\%\)

\(\rho (\text{rho}) = \) correlation coefficient = measures comovement between 2 securities

ex. Individual expected return and standard deviation

<table>
<thead>
<tr>
<th>prob</th>
<th>MGM</th>
<th>STN</th>
</tr>
</thead>
<tbody>
<tr>
<td>.25</td>
<td>3%</td>
<td>-3%</td>
</tr>
<tr>
<td>.50</td>
<td>9%</td>
<td>12%</td>
</tr>
<tr>
<td>.25</td>
<td>15%</td>
<td>27%</td>
</tr>
<tr>
<td>(\hat{r}_i)</td>
<td>9%</td>
<td>12%</td>
</tr>
<tr>
<td>(\sigma_i)</td>
<td>4.24%</td>
<td>10.6%</td>
</tr>
</tbody>
</table>

What does standard deviation tell us?
Portfolio expected return and standard deviation

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>Stock</th>
<th>$r_i$</th>
<th>$\delta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40%</td>
<td>MGM</td>
<td>9%</td>
<td>4.24%</td>
</tr>
<tr>
<td>60%</td>
<td>STN</td>
<td>12%</td>
<td>10.6%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\hat{\kappa}_p$</th>
<th>10.8%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_p$, if $p=+1$</td>
<td>8.0%</td>
</tr>
<tr>
<td>$\delta_p$, if $p=0$</td>
<td>6.6%</td>
</tr>
<tr>
<td>$\delta_p$, if $p=-1$</td>
<td>4.7%</td>
</tr>
</tbody>
</table>

Portfolio Expected Return:

\[ \hat{r}_p = (.40)(.09) + (.60)(.12) = 10.8\% \]

Portfolio Standard Deviation:

say $p=1$, perfect positive correlation

\[ \delta_p = \sqrt{[(.4)^2(4.24)^2 + (.6)^2(10.6)^2 + 2(.4)(.6)(1)(4.24)(10.6)]^{1/2}} \]

\[ = \sqrt{[64.9]}^{1/2} = 8\% \]

$p=0$, no correlation, $\delta_p = 6.6\%$

$p=-1$, perfect negative correlation, $\delta_p = 4.7\%$

**Remember**: less correlation equates to lower risk!!

---

### III. Efficient Portfolios -

the portfolio that provides the highest return for a given level of risk - or lowest risk for a particular expected return.

Therefore combine assets in a portfolio to get highest expected return for given risk ($\delta_p$)

For each asset some risk can be eliminated when combined with other assets in a portfolio (unless $p=+1$)

Combine assets in such a manner to get **Efficient Portfolio**.
Look at an individual stock

Total Risk = $6_i$

Some risk can be eliminated by including the stock in a portfolio - call this that can be eliminated **diversifiable** or **company-specific** risk.

Some risk can not be eliminated - call this **nondiversifiable** or **market** risk.

Risk that is important to investors is **nondiversifiable** or **market** risk.

**risk aversion** (def) - dislike risk

---

### IV. Beta - CAPM

**Beta** = Measure of Market Risk
- the risk that is relevant to investors.

**BETA** ($\beta$) measures of a particular stock's variation in return relative to the market.

\[
\beta = \frac{\text{cov}(k_i, k_m)}{\sigma^2_m} = \frac{p_{im} \sigma_{im}}{\sigma^2_m} = p_{im} \left( \frac{\sigma_i}{\sigma_m} \right)
\]

- $\beta = 1.0$ moves exactly with market
- $\beta > 1.0$ moves more than market (> risk)
- $\beta < 1.0$ moves less than market (< risk)
- $\beta = 0.0$ no risk (Risk-Free)

**CAPM = Capital Asset Pricing Model**

\[
\hat{r}_i = \hat{r}_{RF} + \beta_i (\hat{r}_m - \hat{r}_{RF})
\]

$\left(\hat{r}_m, \hat{r}_{RF}\right)$ = market risk premium

ex. T-Bill = $\hat{r}_{RF} = 5\%, \hat{r}_m = 12\%$

For T-Bill $\beta=0$, no risk

\[
\hat{r}_{\text{T-Bill}} = 5\% + (0)(12\%-6\%) = 5\%
\]

For IBM $\beta=0.7$

\[
\hat{r}_{\text{IBM}} = 5\% + (0.7)(12\%-5\%) = 9.9\%
\]
What does the CAPM tell us?

What is our expected return at a given level of risk (the risk that is important to an investor holding a well-diversified portfolio.

SML = Security Market Line
How many stocks (assets) do we need to hold to approximate a well-diversified portfolio?

Hold everything - market portfolio - eliminates all diversifiable risk - Impossible!

Hold 8-10 assets - closely approximates market (eliminates most diversifiable risk)

Holding a Portfolio - Market Risk (nondiversifiable risk)

Efficient Markets Hypothesis

(def) securities are fairly priced - market price reflects all publicly available information.

What does this mean for investors?

\[ P_0 \quad = \quad \text{fair price according to public information} \]

\[ r_i \quad = \quad \text{depends on risk relative to market risk (} \beta \text{)} \]

Just buy securities and form your portfolio according to risk preference.
Recap: Risk vs. Return

**Goal:** to quantify risk and return so we can compare and choose investment opportunities.

We saw how to calculate expected return

\[ \hat{r} = \sum p_i r_i \]

and risk

\[ \sigma = \left[ \sum (r_i - \hat{r})^2 p_i \right]^{1/2} \]

But if we form a portfolio we can eliminate some risk - if we form portfolio in such a manner to eliminate all extra (diversifiable) risk we have efficient portfolio (def).

---

Assumes that everyone (investors) are bright enough to hold a well-diversified efficient portfolio.

Only risk that matters is nondiversifiable risk (market risk) - measured by \( \beta \)

Use CAPM and \( \beta \) to calculate expected return for relevant risk.

Risk and Return are quantified!!!!

Therefore investors want to own efficient portfolios. Which ones? The one that correspond to the level of risk they want to assume.

( trade-off: **Risk - Return** )