Chapter 8
Risk and Rates of Return

Risk and Return

Which is better?
1. 4% return with no risk, or
2. 15% return with risk.

Cannot say - need to know how much risk comes with the 15% return.

What do we know so far?
We know why returns vary between different securities!

\[ r_{\text{nom}} = r^* + \text{IP} + \text{DRP} + \text{LP} + \text{MRP} \]

- \( r_{\text{nom}} \) = nominal rate
- \( r^* \) = real rate (pure compensation)
- \text{IRP} = inflation-risk premium (change in cost of goods)
- \text{DRP} = default-risk premium (ability to pay P & I)
- \text{LP} = liquidity premium (ability to convert to cash)
- \text{MP} = maturity risk premium (\( \Delta P / \Delta i \))

where \( r^* + \text{IP} = r_{RF} \)

But this does not quantify risk

We want to quantify risk.

Today's concepts
1. quantify risk
2. tells us how much return we need for a given level of risk
Risk and Return - individual assets

Risk = P( Actual returns < Expected returns )

Example: A 2 year Treasury bond; hold until maturity; pays 4% / year

Expected return = 4% [ E(r) = 4% ]
If hold until maturity
Actual return = 4% [ r = 4% ]
Therefore = E(r) = r or P [ r < E(r) ] = 0% no risk!

Risk and Return - individual assets

Buy a 20 year STN bond to hold for 2 years where the expected return = 15% [ E(r) = 15% ]

<table>
<thead>
<tr>
<th>Probability</th>
<th>Actual Return</th>
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</thead>
<tbody>
<tr>
<td>50%</td>
<td>0%</td>
</tr>
<tr>
<td>25%</td>
<td>15%</td>
</tr>
<tr>
<td>25%</td>
<td>45%</td>
</tr>
</tbody>
</table>

Calculating Expected Return

E(r) = Σ ( probi x ri )
= (50% x 0%) + (25% x 15%) + (25% x 45%)
= (0%) + (3.75%) + (11.25%)
= 15.00%

How to measure risk and return

E(r) = expected rate of return

σ = standard deviation which measures the dispersion around the mean (measure of risk)

σ = { Σ [ri – E(r)]² x probi }^{1/2}
Risk and Return

<table>
<thead>
<tr>
<th>Instrument</th>
<th>E(r)</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-Bond</td>
<td>4%</td>
<td>0%</td>
</tr>
<tr>
<td>MGM Bond</td>
<td>15%</td>
<td>18.37%</td>
</tr>
</tbody>
</table>

Problem

Stock A has the following probability distribution of expected returns:

<table>
<thead>
<tr>
<th>Probability</th>
<th>Rate of Return</th>
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<tbody>
<tr>
<td>0.1</td>
<td>-15%</td>
</tr>
<tr>
<td>0.2</td>
<td>0%</td>
</tr>
<tr>
<td>0.4</td>
<td>5%</td>
</tr>
<tr>
<td>0.2</td>
<td>10%</td>
</tr>
<tr>
<td>0.1</td>
<td>25%</td>
</tr>
</tbody>
</table>

What is Stock A's expected rate of return and standard deviation?

Expected Rate of Return

\[
E(r) = \sum (\text{prob}_i \times r_i)
\]

\[
= (.1 \times -15\%) + (.2 \times 0\%) + (.4 \times 5\%)
+ (.2 \times 10\%) + (.1 \times 25\%)
\]

\[
= (-1.5\%) + (0\%) + (2\%) + (2\%) + (2.5\%)
\]

\[
E(r) = 5\%
\]

Standard Deviation

\[
\sigma = \left\{ \sum [r_i - E(r)]^2 \times \text{prob}_i \right\}^{1/2}
\]

\[
= \left\{ [(-15\% - 5\%)^2 \times .1] + [(.05\% - 5\%)^2 \times .2]
+ [(.05\% - 5\%)^2 \times .4] + [(.10\% - 5\%)^2 \times .2]
+ [(.25\% - 5\%)^2 \times .4] \right\}^{1/2}
\]

\[
= \left\{ .004 + .0005 + .0005 + .004 \right\}^{1/2}
\]

\[
= .009^{1/2}
\]

\[
\sigma = 9.49\%
\]
Normal Distribution

Risk and Return for portfolios

Expected Portfolio Return

$$E(r)_p = \Sigma[x_i \times E(r)_i]$$

$x_i =$ proportion of portfolio invested in asset $i$.

Portfolio standard deviation

$$\sigma_p = [x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_Ax_B\rho_{AB}\sigma_A\sigma_B]^{1/2}$$

where, $x_A + x_B = 100\%$

$\rho$(rho) = correlation coefficient

= measures comovement between 2 securities

<table>
<thead>
<tr>
<th>Prob</th>
<th>MGM</th>
<th>LAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>.25</td>
<td>-3.0%</td>
<td>3.0%</td>
</tr>
<tr>
<td>.50</td>
<td>12.0%</td>
<td>9.0%</td>
</tr>
<tr>
<td>.25</td>
<td>27.0%</td>
<td>15.0%</td>
</tr>
<tr>
<td>$E(r)_i$</td>
<td>12.0%</td>
<td>9.0%</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>10.60%</td>
<td>4.24%</td>
</tr>
</tbody>
</table>
Portfolio expected return and standard deviation

<table>
<thead>
<tr>
<th>$X_i$</th>
<th>Stock</th>
<th>$E(r)_i$</th>
<th>$\sigma_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.0%</td>
<td>LAS</td>
<td>9.0%</td>
<td>4.24%</td>
</tr>
<tr>
<td>60.0%</td>
<td>MGM</td>
<td>12.0%</td>
<td>10.60%</td>
</tr>
</tbody>
</table>

Calculations

Portfolio Expected Return

$$E(r)_p = (.40)(.09) + (.60)(.12) = 10.8\%$$

Calculations

Portfolio Standard Deviation

say $\rho = 1$, perfect positive correlation

$$\sigma_p = \left[ (.4)^2(4.24)^2 + (.6)^2(10.6)^2 + (.4)(.6)(1)(4.24)(10.6) \right]^{1/2} = 8.0\%$$

$\rho = 0$, no correlation, $\sigma_p = 6.6\%$

$\rho = -1$, perfect negative correlation, $\sigma_p = 4.7\%$

Remember: less correlation equates to lower risk!!

Efficient Portfolios

The portfolio that provides the highest return for a given level of risk - or lowest risk for a particular expected return.

Therefore combine assets in a portfolio to get highest expected return for given risk ($\sigma_p$)

For each asset some risk can be eliminated when combined with other assets in a portfolio (unless $\rho=+1$)

Combine assets in such a manner to get Efficient Portfolio.
Look at an individual stock

Total Risk = $\sigma_i$

Some risk can be eliminated by including the stock in a portfolio - call this that can be eliminated **diversifiable** or **company-specific** risk.

Some risk can not be eliminated - call this **nondiversifiable** or **market** risk.

Risk that is important to investors is **nondiversifiable** or **market** risk.

**risk aversion** (def) - dislike risk

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**Beta - CAPM**

Beta = Measure of Market Risk - the risk that is relevant to investors

BETA ($\beta$) measures of a particular stock's variation in return relative to the market.  

$$\beta = \frac{\text{cov}(r_i, r_m)}{\sigma^2_m} = \rho_{mi} \sigma_i \sigma_m / \sigma^2_m = \rho_m x (\sigma_i / \sigma_m )$$

- $\beta = 1.0$ moves exactly with market
- $\beta > 1.0$ moves more than market (> risk)
- $\beta < 1.0$ moves less than market (< risk)
- $\beta = 0.0$ no risk (Risk-Free)

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**Capital Asset Pricing Model (CAPM)**

$$r_i = r_{RF} + \beta_i (r_m - r_{RF})$$

Where $(r_m - r_{RF})$ = market risk premium

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**CAPM Example**

Treasury Bill = $r_{RF} = 5\%$

Market Rate = $r_m = 12\%$

For a T-Bill $\beta=0$, no risk

$$r_{T-Bill} = 5\% + (0.0)(12\% - 5\%) = 5\%$$

For IBM $\beta=0.7$

$$r_{IBM} = 5\% + (0.7)(12\% - 5\%) = 9.9\%$$
What does the CAPM tell us?

What is our expected return at a given level of risk (the risk that is important to an investor holding a well-diversified portfolio)

Diversified Portfolio

- How many stocks (assets) do we need to hold to approximate a well-diversified portfolio?
- Hold everything - market portfolio - eliminates all diversifiable risk - Impossible!
- Hold 8-10 assets - closely approximates market (eliminates most diversifiable risk)

Efficient Markets Hypothesis

Efficient Markets Hypothesis (def) - securities are fairly priced - market price reflects all publicly available information.

What does this mean for investors?

\[ P_0 = \text{fair price according to public information} \]
\[ r_i = \text{depends on risk relative to market risk (} \beta \text{)} \]

Says, buy securities and form your portfolio according to risk preference
Recap: Risk and Return

**Goal:** to quantify risk and return so we can compare and choose investment opportunities.

We saw how to calculate expected return

\[ E(r) = \sum (p_i \times r_i) \]

We saw how to calculate risk

\[ \sigma = \left( \sum (r_i - E(r))^2 p_i \right)^{1/2} \]

But if we form a portfolio we can eliminate some risk - if we form portfolio in such a manner to eliminate all extra (diversifiable) risk we have efficient portfolio (def).

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Recap: Risk and Return

Assumes that everyone (investors) are bright enough to hold a well-diversified efficient portfolio.

Only risk that matters is nondiversifiable risk (market risk) - measured by \( \beta \)

Use CAPM and \( \beta \) to calculate expected return for relevant risk.

Risk and Return are quantified!!!!

Therefore investors want to own efficient portfolios. Which ones? The one that correspond to the level of risk they want to assume.

Trade-off: Between Risk and Return