Week Seven

I. **Aquifer characteristics**
   
   A. transmissivity (T)
      1. product of vertical aquifer thickness and K
      2. $T = Kb$
      3. units of $L^2/T$
      4. often more useful than K
      5. carries implicit assumption that flow is horizontal
   
   B. storativity (S)
      1. S volume of water stored or released from the aquifer beneath a unit square of surface area per unit drop in head
      2. dimensionless
      3. specific storage
   
   C. specific storage (Ss)
      1. amount of water released from a unit volume of aquifer material per unit drop in head due to compression of the aquifer and water within
      
      $$S_s = \rho_w g (\alpha + n\beta)$$
      2. $\alpha$ – aquifer compressibility
      3. $\beta$ – water compressibility
      3. $Ss = S/b$
      4. aquifer compression occurs in response to a change in effective stress
         a) water pressure supports the overlying materials
         b) lower the pressure and stress increases
   
   D. Specific yield (Sy)
      1. fractional volume of water released from an aquifer under gravity drainage
      
      $$n_e = Sy + Sr$$ where Sr is specific retention, the water held back against gravity drainage
   
   E. confined vs. unconfined aquifers
      1. confined aquifers generally release water through only $Ss$
   
   F. release from an unconfined aquifer is $Sy + Ss$, much larger

II. **General Overview of Flow to Wells**
   
   A. uses
      1. produce water from the ground
         a) agriculture
         b) municipal
         c) industrial
            (1) processing
            (2) cooling
d) environmental restoration

e) ground control (dewatering)

2. inject water into the ground
   a) waste disposal
   b) artificial recharge
      (1) aquifers
      (2) geothermal
   c) ground control

B. basic concepts

1. drawdown
   a) decline in head at any given location
   b) results from pumping
   c) expressed either as \((h_0-h)\) or \((\Delta s)\)
   d) drawdown is inherently positive

2. cone of depression
   a) water will flow radially to a well
      (1) cross section area decreases near well
      (2) if \(Q\) is constant, gradient must increase
   b) drawdown is zero at large distance from the well
   c) drawdown is maximum at the well bore
   d) drawdown exhibits radial symmetry
      (1) homogeneous, isotropic system
   e) cone of depression in the piezometric surface

3. transient conditions
   a) pump creates a local decline in head
   b) potentiometric surface changes in response
   c) flow is inherently unsteady
   d) steady flow only occurs if aquifer is recharged
      (1) unconfined aquifer
         (a) direct recharge
         (b) intersection of cone with water body
      (2) confined aquifer
         (a) leakage through confining layer
         (b) intersection of cone with water body

III. Basic assumptions

A. well flow equations are very complex
   1. simplifying assumptions are required to reach a solution
   2. solutions are obtained by analogy to heat flow theory

B. different scenarios require different assumptions

C. all scenarios require the following minimal assumptions
   1. aquifer is bounded on the bottom by a confining layer
   2. all formations are horizontal and infinite
   3. potentiometric surface is horizontal prior to pumping
   4. potentiometric surface is at steady-state prior to pumping
5. all changes in head are a result of pumping alone
6. aquifer is homogeneous and isotropic
7. all flow is radial towards the well
8. flow is horizontal
9. Darcy's Law is valid all the way to the well bore
10. water is constant density and viscosity
11. pumping and observation wells are fully penetrating
12. pumping well is of infinitesimal diameter
13. pumping well is 100% efficient

D. the preceding list are mathematical necessities
E. when we use them we must be aware of inconsistencies

IV. Steady-State Flow - Theim Equation

Imagine an island in a lake, and a confined aquifer on the island that meets all of the assumptions discussed above.

In such a system, steady-state flow conditions can exist. In practical terms this situation occurs when the cone of depression encounters sufficient recharge sources to stop draining the aquifer by compression and simply transmits fluid. Also, if a well has been running sufficiently long that there is no meaningful change in head at the observation wells

Problem is solved by writing Darcy's law for flow through an annular region of area $2\pi rb$ and integrating the equation for heads $h_1$ and $h_2$ located on a radial line out from the well

$$Q = (2\pi rb)K \frac{dh}{dr}$$

$$\int_{h_1}^{h_2} dh = \frac{Q}{2\pi T} \int_r^{r_2} \frac{dr}{r}$$

$$T = \frac{Q}{2\pi (h_2 - h_1)} \ln \left( \frac{r_2}{r_1} \right)$$

the Thiem equation can not be used to calculate Storativity because steady-state flow is assumed and Storativity is defined as release of water due to a drop in head at a given location (i.e., transient flow)

V. Steady Flow in an Unconfined Aquifer

By invoking the Dupuit-Forscheimer assumptions to an unconfined aquifer on our circular island the following equations can be developed, of course under these assumptions no recharge is allowed
\[ Q = (2\pi rh)K \frac{dh}{dr} \]

\[ \int_{h_1}^{h_2} h \, dh = \frac{Q}{2\pi K} \int_{r_1}^{r_2} \frac{dr}{r} \]

\[ K = \frac{Q}{\pi (h_2^2 - h_1^2)} \ln \left( \frac{r_2}{r_1} \right) \]

Again this equation cannot be used to predict any time dependent parameters of the aquifer.

**VI. Unsteady radial flow**

From the assumption of a homogeneous and isotropic aquifer we can assume that drawdown around the well will be radially symmetric in such cases it is convenient to express the problem in terms of polar coordinates (theta and radius) rather than x,y velocities must increase as the well bore is approached, hence gradient must also increase, therefore the cone must be a curved surface, note that because of increased velocity, solutions become unrealistic in the immediate vicinity of the well bore

Pumping a fully confined aquifer at steady Q creates a drawdown that never reaches steady-state as water is produced by compressing the aquifer (see section 4.9 for review) rather than recharge. In order to keep squeezing the aquifer, it is necessary to keep dropping the head and the cone of depression continually expands

Such flow is described by the following differential equation

\[ \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t} \]

If the upper confining layer "leaks" the differential equation must be modified accordingly.

\[ \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + \frac{e}{T} = \frac{S}{T} \frac{\partial h}{\partial t} \]

where e describes the vertical leakage.
It is possible for the leaky system to reach a steady state condition where sufficient fluid passes through the leaky layer to satisfy the pumping rate, at such time aquifer compression ceases.

remember that these equations result from the shopping list of assumptions discussed earlier; solving them requires additional assumptions and the use of esoteric mathematical techniques including LaPlace transforms, Bessel Functions, Error functions, and Fourier transforms

solutions to these equations can be used to predict drawdown for a known aquifer; or to solve the inverse problem of determining aquifer parameters from a pump test

**Theis Flow**

additional assumptions

1. Single aquifer is confined top and bottom
2. No recharge
3. Aquifer is compressible
4. water is released instantaneously in response to drawdown
5. Pumped rate is constant

Theis developed the following equation through analogy to heat flow problems (flow of heat in an infinite slab to a perpendicular line sink). Many analytic solutions reported in the groundwater literature were developed in a similar manner, see Carslaw and Jaeger.

\[ h_0 - h = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u} \, du \quad \text{where: } u = \frac{r^2 S}{4Tt} \]

There is no closed form solution to the integral in this equation, however it may be closely approximated by an infinite series.

\[
\int_u^\infty \frac{e^{-u}}{u} \, du \approx -0.5772 - \ln u + u \left( \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} \right) \cdots - \frac{u^{2n}}{(2n) \cdot (2n)!} + \frac{u^{2n+1}}{(2n+1) \cdot (2n+1)}
\]

The infinite series approximation was defined by Theis to be a "Well Function" \( W(u) \) such that:

\[ h_0 - h = \frac{Q}{4\pi T} W(u) \quad \text{for: } u = \frac{r^2 S}{4Tt} \]
Values for $W(u)$ for solution of Theis problems (there are others) are found in tabular form in the text and elsewhere. One would then interpolate between the tabular values if necessary. With the advent of digital computers it is trivial to compute $W(u)$ for any given value of $u$ by using the series approximation and a spreadsheet program.

It is important to note that $W(u)$ is a unique solution (a single curve). This implies that in a Theis aquifer, all potential cones of depression have a shape that is related to this single curve by the other parameters ($S, T, Q, t, r$).

For a given aquifer, $S$ & $T$ will be constant, hence $u$ and $W(u)$ will be functions of distance from the well bore ($r$) and time since pumping initiated ($t$).

Note that the inverse solution, getting $T$ and $S$ from a pump test ($Q, t, r, h_0 - h$ are known) is not directly possible from the Theis equation, as the solution is non-unique; it is also sensitive to small errors in Storativity.
Leaky Aquifer Solutions

The governing equation for flow to a well beneath a leaky confining layer of thickness $b'$ and hydraulic conductivity $K'$ is as follows:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + \frac{(h_0 - h)K'}{T b'} = \frac{S}{T} \frac{\partial h}{\partial t}$$

Hantush developed a solution for the case where

- there is no change of storage (no compression) in the horizontal leaky layer,
- no effective drainage of the overlying unconfined aquifer ($K''$, $b''$)
- unconfined aquifer has a horizontal potentiometric surface.

$$h_0 - h = \frac{Q}{4\pi T} W\left(u, \frac{r}{B}\right)$$

for:

$$u = \frac{r^2 S}{4Tt}, \quad B = \left(\frac{T b'}{K'}\right)^{1/2}$$

Solutions for $W(u, r/B$) can also be found in tabular form.

Since the conditions for this equation to be valid are incredibly restrictive, a series of tests were developed to check "good enough"

$$t < \frac{S'(b')^2}{10bK'}$$

Eq. is OK at short times and/or thick leaky layers

or

$$b''K'' > 100bK$$

EQ. is OK if WT aquifer is real big with respect to pumped aquifer

and

$$t > 0.036 \frac{b'S'}{K'} \text{ Release of water from leaky layer is minimal after it is drained}$$

$$r < 0.04b \left[ \frac{KS_t}{K'S_t} \right]^{1/2} \text{ distance from the well is sufficiently small}$$

$$\frac{r_w}{\left(\frac{T b'}{K}\right)^{1/2}} < 0.1 \text{ Well bore is close enough to infinitesimal}$$
\[ t > \left( \frac{30r_w^2}{T} \right) \left[ 1 - \left( \frac{10r_w}{b} \right)^2 \right] \]

enough time has elapsed so the well bore appears small
Unconfined aquifer

Governing equation is

$$K_r \frac{\partial^2 h}{\partial r^2} + \frac{K_r}{r} \frac{\partial h}{\partial r} + K_v \frac{\partial^2 h}{\partial z^2} = S_v \frac{\partial h}{\partial t}$$

At any given point in the aquifer, there are three phases in the development of the cone of depression

1. In response to initial pressure drop the aquifer responds like a confined aquifer, the skeleton compresses, water expands, and flow is horizontal

2. The water table drops and vertical flow comes into play

3. The water table stabilizes locally and flow is essentially passing through it, horizontally

These three stages occur at different times for each annular region surrounding the well and the system never truly stabilizes. However, at late time the radius of the cone of depression is so large that vertical gradients are sufficiently small to be ignored.

The Neuman solution can be applied at early and very late times.

$$h_0 - h = \frac{Q}{4\pi T} W(u_a, u_b, \Gamma)$$

$$u_a = \frac{r^2 S}{4Tt} \quad \text{early time data}$$

$$u_b = \frac{r^2 S_v}{4Tt} \quad \text{very late time}$$

$$\Gamma = \frac{r^3 K_v}{b^2 K_h}$$

$$K_v, K_h - \text{vertical, and horizontal hydraulic conductivity}$$

$$b - \text{initial saturated thickness of the aquifer}$$
Analysis of aquifer test data

None of the above solutions can be directly inverted to solve for aquifer parameters from time-drawdown data so "tricks" have been developed; in addition to the previous assumptions we have.

1. pumping and observation wells are screened only in the aquifer being tested
2. screens fully cover the aquifer thickness

Theis Non-equilibrium Method

The Theis equation

\[ h_0 - h = \frac{Q}{4\pi T} W(u) \]

can be re-written to yield T and S

\[ T = \frac{Q}{4\pi(h_0 - h)} W(u) \text{ and } S = \frac{4Tut}{r^2} \]

Remember however, that both T and S are typically unknown.

When you perform an aquifer test you have 1 or more observation wells at distances r from the pumping well in which the drawdown is measured as a function of time during a pumped interval at constant(?!) Q. Theis also developed a method for obtaining the aquifer parameters from this data using a technique known as curve matching which follows the following steps.

1. Obtain a standard Theis Type Curve; a plot of W(u) as a function of 1/u on 3 x 5 cycle log-log paper.
2. Plot field drawdown data as a function of time on the same type of paper
3. Tape the type curve onto a light table, place the plotted field data on top, then attempt to match the shape of the field data to the type curve solely by moving the field data parallel to the axes (no twisting).
4. Select a convenient "match point", since it is not required that it coincide with either the field data or W(u) on the type curve, it is best to select a convenient value on the type curve (i.e., W(u) = 1, 1/u = 1). Then measure values for t and drawdown on the field data graph that correspond to the match point.
5. Use the match point values to calculate T, then S
Jacob Straight Line Method

Jacob noted that at large \( t \), the variable \( u \) in the Theis equation becomes small. At small \( u \) (\( u < 0.05 \)) the higher order terms in the infinite series approximation become negligible.

\[
\int_{u}^{\infty} \frac{e^{-u}}{u} \, du = -0.5772 - \ln(u) + \frac{u^2}{2 \cdot 2!} - \frac{u^3}{3 \cdot 3!} + \frac{u^4}{4 \cdot 4!} - \ldots - \frac{u^{2n}}{(2n) \cdot (2n)!} + \frac{u^{2n+1}}{(2n+1) \cdot (2n+1)!}.
\]

\[
\int_{u}^{\infty} \frac{e^{-u}}{u} \, du = -0.5772 - \ln u
\]

\[
T = \frac{2.3Q}{4\pi(h_0 - h)} \log \left( \frac{2.25Tt}{r^2S} \right)
\]

Again \( T \) and \( S \) cannot be separated, so a trick is employed; the log term is split.

\[
T = \frac{2.3Q}{4\pi(h_0 - h)} \left[ \log \left( \frac{2.25T}{S} \right) + \log \left( \frac{t}{r^2} \right) \right]
\]

then either \( t \) or \( r^2 \) is held constant and the drawdown is differentiated with respect to the variable; by integrating over one log cycle the of \( t \) or \( r^2 \) then it drops out of the equation and we get the solution shown below.

\[
T = \frac{2.3Q}{4\pi \Delta (h_0 - h)}
\]

\[
S = \frac{2.25Tt_0}{r^2}
\]

where

\( t_0 \) is the time were the straightline intercepts zero drawdown

Drawdown is plotted as a function of time on semi-log paper. If the Jacob method is applicable, this data will plot (mostly) on a straight line. The change in drawdown over one complete log cycle is then measured and defined as \( \Delta (h_0 - h) \) and \( t_0 \) is defined as the intercept of the straight line with drawdown = 0

As with all of these methods length can be in feet or meters, but all time units have to be in days.
**Unconfined aquifers**

This is also a type curve matching problem

1. match the early time data first, and mark a match point
2. match the late time data staying on the same curve
3. remember there are an infinite number of intermediate curves

please note that this is truly miserable

**Slug test**

Aquifer tests are:

- expensive, time consuming
- contaminated water may be produced
- don't work real well in low K materials

slug and bail down tests - increase/decrease head in a well

- add/remove water
- drop in a solid slug

recovery of the well is plotted as a function of time

this only samples K in the immediate vicinity of the hole, be aware of disturbed zones, geometry of the hole is very important

**Linear superposition**

the LaPlace equation for confined aquifers is linear, meaning that solutions are additive

if we want to consider more than one well we only need to add the drawdowns at each point in the aquifer

can also consider the effects of uniform flow

this process can also be used to deal with hydrologic boundaries through the use of image wells

- recharge well for constant head boundary
- discharge well for a no-flow boundary

boundaries are assumed to be fully penetrating, vertical, and infinitely thin