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Dear Professor Choi:  

I enclose a referee's report on your paper, coauthored with Jameson, entitled "Lookback Option Valuation: A Simplified Approach," that was submitted to the Journal of Derivatives (MS 22083). I have also read it carefully myself several times. Both the referee and I like the paper, but there are a number of ways in which it can be improved. 

The referee makes a number of comments and suggestions. It strikes me that one or more of the points in the report may not actually be correct (is the problem for which he shows a diagram right?). I'm not sure, and this is one of the problems in the paper. It needs to be made clearer, so that a careful reader can understand it fully without a lot of trouble. After several careful readings, I'm still not sure I know completely how it works, even though, as a mechanical algorithm, it is not that complicated. 

Here are some suggestions.  

Introducing both fixed and floating strike options and talking about them at the same time on p.6 is confusing. Start with one example (the floating strike) and take us all the way through it. Then quickly show the fixed strike case. Then proceed to the Tables 1 and 2 examples. 

Figure 1 confused me a lot. In looking at this diagram, I was expecting it to be a standard lattice, with the time step on the horizontal. Please walk us slowly through this, clarifying that this is a single time slice. Show the prices on the vertical axis, too. 

One thing that makes the presentation more difficult is that the origin for the price lattice is simply some arbitrary $S_0$. It would be much easier to follow if the origin were $S_0$ and the prices at later dates were shown as $S_N, ... , S_i, S_0, S_1, ... , S_N$. 

July 22, 2003
How to handle a max or min different from the initial $S_0$ value, mentioned in footnote 5, should be shown explicitly, possibly in the main text. If it is involved, leave it in the footnote but the reference in the text should make it clear that this is in footnote 5 (the JOD uses endnotes, which are even less likely to be read unless they are specifically pointed out).

Pp.8-9 is really the heart of what you are doing, but it is really hard to follow. It's already confusing to have notation with time running both forward and backward, though that is perhaps simpler than an alternative. I actually think the best way to clarify all of this would be to put in a very simple numerical example at this point. Show us how things would actually work in, say, a specific 4-step lattice, giving the exact calculations for some $m = 1$ and $m = 2$ nodes. Maybe a little table would help? I'm not sure what the best way to do this would be, but I do think the essence of what is needed here is to show the reader intuitively and specifically what is going on. Figures 3 and 4 should not be introduced together. Do the two types of options separately and introduce each figure with the appropriate option.

The point that the values of two ATM options on the same underlying must be proportional is clearly key. Please expand on it a little. Is this not due to the property that an option value is homogeneous of degree 1 in $S/X$. If so, I don't think it is only true for the lognormal. Isn't it in Merton's 1973 Bell Journal paper as a general property options? Restricting your method to only the lognormal is a major limitation on it. Do you have to?

One example of something that wasn't clear to me in the description is whether you need to compute Black-Scholes values for each node in the tree (or at least a lot of them), or is it only at time $T-1$ that you use BS. I'm sure I could puzzle it out, but the point is that I shouldn't have to. The presentation should be clear and complete enough that the answer would be obvious.

There should be an r in the discounting formula following eq. (4).

One very important thing that you need to show explicitly is how to handle the case with discrete monitoring when there are more time steps than monitoring points. How does one handle the price adjustments when the next, or previous, date is not a monitoring time?

Please give more info about the RBF approach. One of the really valuable aspects of your technique is that it is applicable to many different numerical approximation methods. The RBF methodology is useful, but a lot of readers won't quite recall how it works.

A second important thing that needs to be explained more clearly is how one handles the case with multiple sources of risk. You now have, say, $K$ sources of risk, one of which is the asset price, on which the lookback feature is defined. The full-scale lattice would require $K + 2$ dimensions, which you are able to reduce to $K + 1$, by not having to carry
the max (or min) as an extra state variable. Show us at least a token calculation so that it is clear how your method is implemented for this case.

The bottom line is that I like your idea a lot. It is a simple, and I think powerful, approach to improve valuation of an important class of options. But it is tricky to get one's mind around it completely. Your paper needs to make gaining the intuition for what you are doing as easy for the reader as it can possibly be. Strive for clarity and understanding, even at the risk of being a little redundant and oversimplified. Better to be too easy for some readers than too complicated for most. Given the thorough rewriting that I am asking for, you should consider this a Revise and Resubmit, but with the expectation that when you have successfully turned it into a more "reader-friendly" piece, it will be accepted.

Thank you for sending your work to the Journal of Derivatives. I am looking forward to seeing the next version.

Sincerely,

Stephen Figlewski
Editor
LOOKBACK OPTION VALUATION: A SIMPLIFIED APPROACH

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Abstract

In this paper, we introduce a simple method to value lookback option. The approach handles path-dependency without introducing an additional dimension or requiring a transformation of underlying state variable. We show that lookback options can be valued as if they are path-independent standard options. Accordingly, the proposed method would be very useful when one considers valuation of the lookback option with the multiple source of uncertainty.
1. Introduction

Lookback options are options for which the payoff depends on the maximum or minimum underlying asset price reached during the life of the option. Their use has recently become widespread on derivatives markets, particularly over-the-counter currency options markets. The academic literature proposes several numerical valuation methods that accommodate the path dependency of these options. For example, see Hull and White (1993), Cheuk and Vorst (1997) and Wilmott, Howison, and Dewynne (1995). Their numerical approaches deal with the path-dependency feature of lookback options either by explicitly including the historical minimum or maximum underlying asset values as an additional dimension or by tracing the possible minimum or maximum values in the underlying asset price process. These approaches have the drawback that they may become computationally burdensome or difficult to extend to more general cases when there are multiple sources of uncertainty.

In this paper we propose an alternative methodology that accommodates the path dependency features of lookback options simply and naturally. The proposed technique requires neither the additional dimension that may arises from the path-dependency features nor a transformation of underlying state variable as in Cheuk and Vorst (1997). Indeed, we show that well-known “backward” numerical procedures used to value standard options, such as lattice, finite difference, or finite element methods, can be applied to lookback options despite their path dependency. This turns out to be possible because, at each time step, in the backward computation, the option values at some nodes can be replaced by the values readily available in the computation process that capture
the effects of path dependency. Thus, the proposed technique is easily implemented by adding one simple step to any numerical approaches such as lattice or finite difference methods used for standard option valuation. An advantage of the technique is that it may be also applied to numerical approaches for multiple sources of uncertainty without introducing additional complexity.

The balance of the paper explains the approach and presents examples of the results it achieves. Section 2 reviews briefly the lookback option and existing techniques in the literature. Section 3 explains how the path-dependency of lookback options is accommodated without creating an additional dimension or transforming the state variable in the evaluation. Section 4 presents numerical examples for the cases of a single source and multiple sources of uncertainty. Section 5 concludes.

2. Lookback options and existing valuation methods

There are two classes of lookback options: floating strike and fixed strike options. A European lookback floating strike call option gives the holder the right to buy the underlying asset at expiration at the historical minimum price reached during the option's life. A European lookback fixed strike call option pays either zero or the difference between the historical maximum price and the strike price, whichever is greater. Closed form solutions have been found for lookback options of the European type. However, in addition to path-dependency, lookback options commonly exhibit other features that complicate their valuation. Two of these that seem to be particularly
important in practice are discrete monitoring (monitoring to determine historical extreme prices occurs only at discrete, contractually specified intervals) and the existence of multiple sources of uncertainty that affect lookback option value. Examples of the latter include the currency option modeled by Amin and Jarrow (1991) and options written on multiple assets as considered by Stulz (1982). The closed form solutions noted above do not allow for either of these possible complicating features. Instead, they assume the underlying asset price is monitored continuously and is driven by a single source of uncertainty following a geometric Brownian motion.

Suggestions have been raised that address, to one degree or another, each of these problems individually. However, when both occur simultaneously, as is usually the case in practice, these approaches tend to conflict. For example, Broadie, Glasserman and Kou (1997, 1999) introduce correction terms that allow continuous-time valuation formulas to be applied to discretely monitored options driven by a single source of uncertainty. It is not clear, however, how these continuity correction factors can be extended to a multi-dimensional context. Alternatively, Cheuk and Vorst (1997) incorporate discrete monitoring in a numerical approach based on a binomial lattice. They show that the added state variable due to path-dependency of lookback options can

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1 See Goldman, Sosin and Gatto (1979), Garman (1989) and Conze and Viswanathan (1991) for the closed form solution of lookback options.
2 The practical importance of multi-dimensional path-dependent options is underlined by Ritchkin (1995) and Heynen and Kat (1994).
3 In principle analytic values for discretely monitored path-dependent options can be derived based on the multi-variate normal distribution (Heynen and Kat, 1995). However, because the dimensionality of the multi-variate normal distribution required equals the number of monitoring periods, this approach is tractable only for options subject to a very limited number of monitoring times.
be avoided if valuation proceeds after a transformation of the state space.\(^4\) However, there are two difficulties associated with their proposed method. First, it applies only to lattice-type numerical techniques. It is not clear their method can be used with other numerical techniques, such as finite difference methods, that may be more suitable for certain option valuation problems. Secondly, it is unclear, though not impossible, that their transformation technique can be extended to the cases involving multiple source of uncertainty.

3. Valuation for lookback options.

For the sake of simplicity, we consider a currency lookback option valuation problem including only one source of uncertainty and with constant domestic and foreign interest rates. In the risk-neutral world, currency, \(S\), denoted in terms of domestic currency per unit of foreign currency is assumed to follow:

\[
\frac{dS}{S} = (r^d - r^f)dt + \sigma dW
\] (1)

where \(dW\) is a Wiener process, \(r^d\) and \(r^f\) are constant and represent the domestic and foreign risk-free interest rate respectively and \(\sigma\) is the volatility of the continuously compounded rate of return on the underlying asset. The payoffs of lookback options depend on the historical minimum or maximum price reached during the option's life. If we define

\[
J_{t,\tau} = \min_{t \leq \tau} S(\tau)
\]

and

\[^4\] A numerical approach to path dependency, based on Hull and White (1993), compounds the
then a floating-strike lookback call has a payoff at expiry

\[(S(T) - J_T)^+\]

and a fixed-strike lookback call (or lookback rate call) has a payoff at expiry

\[(\bar{J}_T - K)^+\]

where \(K\) is a strike price.

An obvious problem in valuing either kind of lookback option is the dependency on the previous maximum or minimum. To explain how we deal with this path-dependency, we begin with an illustration of the grid that would result if the past extremum is treated as an added state variable (See Figure 1 and 2). We note first that the definition of historical minimum (maximum) immediately eliminates half the grid from consideration since the problem is only valid for \(S(t) \geq J_T\) (\(0 \leq S(t) \leq \bar{J}_t\)) for the floating strike (fixed strike) lookback call option. This permits us to confine attention to grid points below (above) the diagonal line as in Figure 1 and Figure 2.

[Insert Figure 1 and 2 around here]

In order to avoid this additional dimension, we will show that it suffices to consider only the grid points on the solid line indicated in Figure 1 and Figure 2. This means we consider only one grid point for each chosen value of \(S\), removing the extra multidimensionality problem by treating the past extremum as yet another state variable.
dimension. As in Figure 1 and 2, suppose we choose a set of uniform grid points 
$[S_1, S_2, \ldots, S_{j-1}, S_j, S_{j+1}, \ldots, S_N]$ as the computational domain for the underlying asset prices. Further, let $S_j$ represent the underlying asset price at the time of option origination. Since "history" starts with option origination, $S_j$ is also the historical minimum or maximum price at that time.\(^5\) In the case of a floating strike lookback option, whenever the underlying asset price falls below $S_j$ during a monitoring period, the new asset price is also a new historical minimum; if the asset price rises above $S_j$, the historical minimum remains at $S_j$. This is the reason why the solid line is kinked at $S_j$ in Figure 1. Likewise, for the case of a fixed strike lookback option, underlying asset prices above $S_j$ become historical maximum prices themselves and the prices below $S_j$ leave the historical maximum unchanged at $S_j$. This, again, means that the kinked line in Figure 2 is the relevant states to which the system could move from $S_j$. Thus, if only one time step were involved in option valuation, it would be sufficient to consider only points on the solid line. We now present an inductive argument to show that this proposition holds generally.

3.A. Floating strike lookback call

Given an appropriate number of time steps, $M$, such that the uniform time step size is $\Delta \tau = \frac{T}{M}$, we can now apply a simple backward time integration scheme for $m = 0, 1, 2, \ldots, M$ as explained below. For notational convenience, we use the superscript in

\(^5\) If the historical maximum (minimum) price is chosen to be different from the initial asset value, a simple adjustment suffices to correct for this difference.
parenthesis, \((m)\) to mean the number of time steps updated backward from the option expiration date so that, for example, \(V^{(m)}\) implies the option value at time \(T - m \cdot \Delta \tau\).

The inductive argument shows that, at each time step, it is sufficient to consider the case \(J^{(0)} = J_0 = S_j\), that is to say, the case in which the historical minimum at each time step equals the initial asset price. Thus, let us consider the terminal values of the option to be

\[
V_T = V^{(0)} = \left(S(T) - J^{(0)}\right)^+
\]  

(2)

for the chosen mesh points, assuming the historical minimum asset price, \(J^{(0)} (= J_T)\), is \(J_0\). In this case, the payoff is represented by line (1) in Figure 3. These are also the relevant terminal payoffs for use with either the PDE approach or lattice approach to evaluating an option at one time step prior to expiration, i.e., \(m=1\), when it is assumed that \(J^{(1)} = J_0\). If the asset price at expiration exceeds \(J^{(1)}\), \(J_0\) remains the historical minimum and the payoff diagram is correct. Of course, if the asset price at expiration falls below \(J^{(1)}\), then \(J^{(0)} (= J_T)\) is not \(J_0\), but the new asset price (which is now the historical minimum). Nevertheless, the payoff at this lower price is still zero so the payoff diagram is correct in this case, as well.

[Insert Figure 3 and 4 around here]

When this payoff diagram is used to calculate option values at \(m = 1\) by any numerical approach, it yields option values like those represented by line (2) in Figure 3. These are a numerical approximation of a standard call satisfying Equation (1) with strike price \(S_j\). These values are also correct for the floating strike lookback for all \(S \geq S_j\).
However, the assumption that $J^{(1)} = S_j$ cannot be maintained when $S < S_j$, because this asset price is itself a new minimum. But in this case the option is in effect an at-the-money call option since the current asset value equals the historical minimum. This fact can be used to calculate the value at these nodes without resort to future option values. Given the lognormal asset price distribution, the arbitrage-free value of any two at-the-money options on the same underlying asset must be proportional.\(^6\) The value of the remaining nodes can be calculated as a portion of the value of the node, $S_j$, that was at-the-money. Thus, we simply replace the option values by

$$
\frac{S}{S_j} \cdot V(S_j, m) \text{ for all } S < S_j
$$

(3)

where $m = 1$.\(^7\) The replaced option values are represented by line (3) in Figure 3. These are the relevant values for use in evaluating an option at $m = 2$ when it is assumed that $J^{(2)} = L_0$. If the asset price at $m = 1$ exceeds $L_0$, $L_0$ remains the historical minimum and the option values must be consistent with line (2) which is the same as line (3). If the asset price at $m = 1$ falls below $S_j$, a new minimum prevails in which case the option values stays on line (3). Thus the option values of line (3) are now used to update the option values for $m = 2$. The same updating procedure is followed iteratively, until value at the time of issue is calculated. At each step $m$, only one set of option values - those based on the assumption $J^{(m)} = S_j$ need to be calculated. The method works because at

\(^6\) A floating strike lookback call is at-the-money call when the current asset price and the historical minimum are the same.

\(^7\) Of course it would alternatively have been possible to evaluate the lower node points by considering expiration values assuming the new, lower minimum. But the stated substitution approach makes this unnecessary.
each step it is sufficient to calculate values for in-the-money and at-the-money options. The substitution procedure takes care of the case when historical new minimum values (the interior points on Figure 1) would have to be considered.

[Insert Figure 5]

Note that this applies equally to the PDE formulation and the lattice version. For the case of the trinomial approach, as shown in Figure 5 for $m = 1$, all option values above $V^{(i)}_j$ are obtained by the standard risk-neutral valuation technique and the remaining option values are obtained by Formula (3) above.\(^8\)

3.B. Fixed strike lookback call

Essentially the same substitution eliminates the need for an additional state variable when evaluating fixed-strike lookback options. We consider a fixed strike lookback option whose strike price, $K$, is set at the current asset price, $S_j$, which is the historical maximum price at option origination. Again we begin by considering terminal values of the option $V^{(0)}_j = V_T = (\bar{J}^{(0)}_j - K)^+$ for the chosen mesh points on the solid line in Figure 2. The historical maximum asset price at expiration, $\bar{J}^{(0)}_j (= \bar{J}_T)$, is assumed to be $\bar{J}_0$. In this case, the payoff is represented by line (1) in Figure 4. These are the relevant terminal payoffs when it is assumed that $\bar{J}^{(0)}_j = \bar{J}_0$ for use in evaluating an option at one time step prior to expiration, i.e., $m=1$. If the asset price at expiration falls below

\(^8\)The binomial approach presented by Cheuk and Vorst is a special case of the trinomial approach just as the standard binomial approach is a special case of the trinomial approach.
\( \overline{J}^{(0)} \), \( \overline{J}^{(0)} \) remains the historical maximum and the payoff diagram is correct. Of course, if the asset price at expiration exceeds \( S_j \), then \( \overline{J}^{(0)} (= \overline{J}_T) \) is not \( \overline{J}^{(0)} \), but the new asset price (which is now the historical maximum). Accordingly, the payoff at these prices becomes \( S(T) - K \) as indicated in the diagram. Suppose the option values are numerically updated for \( m = 1 \). Let the updated option values be represented by line (2) in Figure 4 such that Equation (1) is approximately satisfied. As in the case of the floating strike lookback call, we distinguish between option values for \( S \geq S_j \) and \( S < S_j \). While the updated option values are relevant for \( S \leq S_j \), a new maximum prevails when \( S > S_j \) and the payoff line (1) at expiration does not apply. Instead, the value of the remaining nodes are calculated as a portion of the value of the node, \( S_j \), that was at-the-money. Setting a new maximum asset price means that holding this security is equivalent to holding an at-the-money call option with the new strike price \( \overline{J}^{(1)} \) and a certain payment of the excess of the maximum to date and the fixed strike (\( \overline{J}^{(1)} - K \)). Thus the values for these stock prices at date 1 are the sum of the value of the call (computed as a proportion of the value of the at-the-money call with strike \( \overline{J}^{(0)} \) used originally - this is analogous to the previous case) and the present value of this fixed sum. That is, for the option values for \( S \geq S_j \), we substitute

\[
\frac{S}{S_j} \cdot V(S_j, m) + PV^{(m)}[S - S_j] = \frac{S}{S_j} \cdot V(S_j, m) + e^{-m\Delta t} \cdot (S - S_j)
\]

where \( PV^{(m)}[S - S_j] = e^{-m\Delta t} \cdot (S - S_j) \) is the present value of \( S - S_j \) at \( m = 1 \). The first term in (4) is depicted by line (3) in Figure 4 and the value including the second term is
depicted by line (4). Thus the option values of line (4) are now used to update the option values for \( m = 2 \). The same updating procedure is followed iteratively for \( m = 2, 3, \ldots, M \).

4. Numerical Examples

In this section, we present numerical examples of currency lookback option valuation for the cases of single and multiple sources of uncertainty. We apply the methodology proposed in Section 3 to two numerical option valuation approaches: the trinomial and the Radial Basis Function (RBF) approach. While the trinomial approach is well known in the option literature, the RBF approach to option valuation is relatively new. Thus, we briefly explain the RBF approach.

As shown in Choi and Marcozzi (2001), the RBF approach, similar to finite-difference methods, approximates the true option value by discretizing the partial differential equation (PDE)\(^9\). But it employs radial basis functions as the basis of the approximation. It requires only that the form of the radial basis function be selected and a mesh generated, independent of the dimensionality or geometry of the problem. The option value is then approximated by a linear combination of the basis functions with time-varying coefficients. Because of its simple requirements, we choose the RBF approach to show how our proposed approach to lookback option pricing works in the cases of single and of multiple sources of uncertainty. Choi and Marcozzi (2001) consider exactly the same numerical example of a plain vanilla currency option valuation problem with stochastic interest rates. Thus, we simply add the lookback features to their
setting. Since the case in Choi and Marcozzi (2001) involves multiple sources of uncertainty, we simplify their model to a single source of uncertainty by assuming domestic and foreign interest rates are constant.

The trinomial approach used for the single source of uncertainty case assumes that state $S$ at time $t$ moves to state $uS$, $S$, or $dS$ at time $t + \Delta t$ with probabilities $p_u$, $p$, and $p_d$, respectively, where

$$u = e^{\lambda \sqrt{\Delta t}}, \quad d = \frac{1}{u}$$

and

$$p_u = \frac{1}{2\lambda^2} + \frac{(r^d - r^f - \sigma^2/2)\sqrt{\Delta t}}{2\lambda \sigma}, \quad p = 1 - \frac{1}{\lambda^2}, \quad \text{and} \quad p_d = 1 - p_u - p.$$  

For $\lambda=1$, $p = 0$, this becomes the binomial approach of Cox, Ross and Rubinstein (1979).

We note, however, that there is one important difference between implementing the binomial tree in the usual way, and implementing it as a trinomial with $\lambda =1$, $p=0$. Although the middle of the three trinomial nodes receives no weight when calculating the value at the next step, the algorithm does calculate a value for each node. In contrast, when the binomial is implemented in the usual way, half the time steps do not have a node at asset price $S_j$, instead, nodes occur one step above and one step below this value. This means that at these time-steps no value is calculated for an option that is exactly at the money. Ordinarily this would not affect the final option value reported. However, for the method used here, in which the computed at-the-money value is substituted into

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9 The performance of RBF approaches has also been examined numerically in Marcozzi, Choi and Chen.
formulas for value at many nodes, it is important for the numerical accuracy of the results to have values computed exactly at-the-money. For this reason, we implement the binomial as a special case of the trinomial.

Tables One and Two report the numerical examples obtained when the above three methods are applied to currency lookback call options featuring, respectively, floating and fixed strike prices. In both cases it is assumed the calls are European, monitoring is continuous, and interest rates are non-stochastic. The specific parameter values used throughout both tables are $S = 100$, $r_d = 0.04$, $r_f = 0.07$, $\sigma = 0.2$, $T = 0.5$. These are the same values employed by Cheuk and Vorst (1997). An analytic solution is available for continuously monitored, European lookback options; the values in this case are 9.792 and 10.977. The first column indicates the number of time steps used to implement the binomial and trinomial calculations. Column two reports the binomial results. (As stated above, these were actually implemented as a trinomial with $\lambda =1$.) These results are identical to those reported by Cheuk and Vorst (1997), verifying the computational equivalence of the two methods in this special case.

The third column reports results for the trinomial model where $\lambda =1.22474$. This gives approximately equal weight to all three nodes in each calculation. Note that in these particular numerical examples, the trinomial consistently underestimates the true value by more than the binomial ($\lambda =0$). This may be because the binomial puts more weight on the in-the-money outcome than does the trinomial in the very first updating procedure. This tends to raise the calculated result because the at-the-money and out-of-
the-money outcomes are worth zero at expiration. This effect assumes magnified importance when the at-the-money value is substituted into the computation of the out-of-the-money values. However it must be diminished when the discrete monitoring case is considered, because the substitution is carried out only at monitoring times.

The final two columns report the results obtained using the RBF method. A potential advantage of this, or any other PDE approach, is that it permits separate determination of the computational domain – the number and location of mesh points – and the number and location of the time steps. For the binomial and trinomial methods, the location and number of mesh points is automatically linked to the number of time steps. If \( N \) is the number of time steps, the traditional binomial approach generates \( N + 1 \) mesh points at expiration, the trinomial \( 2N + 1 \). The resulting computational domain at expiration depends on this number and on the parameter \( \lambda \) through its influence on the step size. In both applications of the RBF method, the computational domain and mesh points are set to coincide with that produced by the binomial method. In column four, the number of time steps used in the calculation also corresponds to the number used in the binomial case. In the final column, the number of time steps is set at six times the number in the binomial case, while maintaining the same computational domain as the binomial. When the number of time steps are the same, the RBF and binomial performance is very close. When the number of time steps is increased, the RBF outperforms the other methods. Nevertheless, as already noted by Cheuk and Vorst (1997), the convergence to the continuous monitoring value is very slow. In fact, if we did not already know the exact value by analytical methods, it would be extremely
difficult to recognize the progression of values as numerical convergence to some constant.

Having shown that the RBF performs as well or better than the alternatives in a case where comparison is readily possible, we now consider its application in cases that are computationally quite burdensome for lattice methods. Specifically, we increase the sources of uncertainty from one to three, adding stochastic domestic and foreign interest rates to the stochastic exchange rate. In addition, we assume that monitoring occurs only at discrete intervals. The results are reported for a floating strike lookback currency option with parameter values used in Choi and Marcozzi (2001)\(^{10}\). Here there is only one case in which we know the exact solution for the lookback option. That is the case of a standard European call option, since the standard option is a special case of the lookback option where the underlying asset value is monitored only once, at expiration. The exact value for a standard call option following this process can be found in Amin and Jarrow (1991) to be 0.0835.

Table Three describes the result of applying the RBF approach to value this option when 125 time steps are used. The computational domain was designed to conform as nearly as possible to those reported in the previous tables. First we calculated the computational domain that would have been generated by a binomial model in which the only source of uncertainty was an exchange rate process with the indicated variance. The range of values for both interest rates was then set to an equal size, with the result

\[\begin{align*}
T &= 0.5, \quad \theta_d = \theta_f = 0.005, \quad \sigma_{xx} = 0.0918, \quad \sigma_{xd} = \sigma_{xf} = 0.0032, \\
\sigma_{dd} = \sigma_{ff} &= 0.0026 \quad \text{and} \quad \sigma_{df} = 0.0005. \\
\end{align*}\]

The results shown in the table are at \(S=1\) (or \(x = \log(S) = 0\)).

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\(^{10}\) We have replicated Choi and Marcozzi (2001) with values \(T = 0.5, \quad \theta_d = \theta_f = 0.005, \quad \sigma_{xx} = 0.0918, \quad \sigma_{xd} = \sigma_{xf} = 0.0032, \quad \sigma_{dd} = \sigma_{ff} = 0.0026 \quad \text{and} \quad \sigma_{df} = 0.0005. \quad \text{The results shown in the table are at } S=1 \quad \text{(or } x = \log(S) = 0)\).
that the computational domain at each time step was a cube. Column one describes how finely the mesh points are set in this cube. For example, $11 \times 5 \times 5$ means that 11, 5 and 5 mesh points were used for exchange rates, domestic interest rates and foreign interest rates respectively. Thus the number of nodes at which values are calculated at each time step ranges from 125 to slightly over 4,000. Experimentation suggested that devoting additional mesh points to the interest rates did not much improve accuracy. For that reason most of the variation reported in the table is with respect to the exchange rate. We note that for the option monitored only at maturity, reported in column two, the results are quite accurate with the mesh set at $21 \times 5 \times 5$. The remaining columns report the results for options that are monitored more frequently. In the final column results are reported for the case in which the option is monitored at every one of the 125 steps, corresponding to an option that is continuously monitored. Although true values are not known for these options, we note that the estimates are well behaved in the sense that, as the number of mesh points increases, they show convergence behavior similar to that observed in Tables One and Two. In particular, the increment of the value decreases as the mesh points increase.

As a final check, Table Four reports the results for the same options using a very large number of time steps, 5,000. The results are reassuring in that they generally show that the extra time steps do not much change the computed values suggesting that convergence was substantially complete with 125 time steps. The continuous monitoring case is slightly more sensitive than the other cases, probably due to the greater number of substitutions resulting from the frequent monitorings.
5. Conclusion

In this paper we introduce a simple numerical methodology for valuing lookback options. The method handles the path dependency of lookback options without increasing the dimension of the computational problem. It also permits the mesh structure and location of the time steps in the calculation to be determined independently. This flexibility and relative parsimony permits convenient application in situations in which alternative approaches become unwieldy, such as options having values dependent on multiple sources of uncertainty or subject to monitoring at discrete times.

The numerical results presented here show that the method performs at least as well as existing methods when applied to relatively simple cases with known analytic solutions (such as European lookback calls with continuous monitoring). The method also handles more complicated cases (such as discretely monitored currency lookback options when the option value depends on multiple sources of uncertainty.)
Table 1. European floating strike currency lookback call

<table>
<thead>
<tr>
<th>N</th>
<th>Binomial model</th>
<th>Trinomial model ($\lambda=1.22474$)</th>
<th>RBF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Time step=N</td>
</tr>
<tr>
<td>50</td>
<td>8.969</td>
<td>8.783</td>
<td>8.752</td>
</tr>
<tr>
<td>100</td>
<td>9.201</td>
<td>9.067</td>
<td>9.045</td>
</tr>
</tbody>
</table>

The parameters used are $S = 100, r_d = 0.04, r_f = 0.07, \sigma = 0.2, T = 0.5$. The Analytic solution for continuously monitored option is 9.792. * The number of mesh points used is 2001.

Table 2. European fixed strike currency lookback call

<table>
<thead>
<tr>
<th>N</th>
<th>Binomial model</th>
<th>Trinomial model</th>
<th>RBF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Time step=N-1</td>
</tr>
<tr>
<td>100</td>
<td>10.027</td>
<td>9.863</td>
<td>9.836</td>
</tr>
<tr>
<td>500</td>
<td>10.427</td>
<td>10.351</td>
<td>10.337</td>
</tr>
<tr>
<td>1000</td>
<td>10.524</td>
<td>10.471</td>
<td>10.460</td>
</tr>
<tr>
<td>5000</td>
<td>10.656</td>
<td>10.632</td>
<td>10.630*</td>
</tr>
</tbody>
</table>

The parameters used are $S = 100, r_d = 0.04, r_f = 0.07, \sigma = 0.2, T = 0.5$. The Analytic solution for continuously monitored option is 10.977. * The number of mesh points used is 2001.
Table 3: European floating strike currency lookback call with stochastic interest rates. (Time step=125)

<table>
<thead>
<tr>
<th>Mesh Points</th>
<th>Monitoring Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>X × R^d × R^f</td>
<td>1</td>
</tr>
<tr>
<td>5×5×5</td>
<td>0.0774</td>
</tr>
<tr>
<td>11×5×5</td>
<td>0.0828</td>
</tr>
<tr>
<td>21×5×5</td>
<td>0.830</td>
</tr>
<tr>
<td>41×7×7</td>
<td>0.0831</td>
</tr>
<tr>
<td>51×9×9</td>
<td>0.0831</td>
</tr>
</tbody>
</table>

The reported numerical results are the option values evaluated for at-the-money option when S=1 (x = \log(S) = 0). The parameters associated with the stochastic processes of the variables can be found in Choi and Marcozzi (2001). The values used are: T = 0.5, \Theta^d = \Theta^f = 0.005, \sigma_{xx} = 0.0918, \sigma_{xdl} = \sigma_{xdr} = 0.0032, \sigma_{dd} = \sigma_{ff} = 0.0026, and \sigma_{df} = 0.0005. The exact European standard call option value is 0.0835, in which monitoring occurs only once (column 2).

Table 4: European floating strike currency lookback call with stochastic interest rates. (Time step=5000)

<table>
<thead>
<tr>
<th>Mesh Points</th>
<th>Monitoring Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>X × R^d × R^f</td>
<td>1</td>
</tr>
<tr>
<td>5×5×5</td>
<td>0.0775</td>
</tr>
<tr>
<td>11×5×5</td>
<td>0.0829</td>
</tr>
<tr>
<td>21×5×5</td>
<td>0.0831</td>
</tr>
<tr>
<td>41×7×7</td>
<td>0.0831</td>
</tr>
<tr>
<td>51×9×9</td>
<td>0.0831</td>
</tr>
</tbody>
</table>

The reported numerical results are the option values evaluated for at-the-money option when S=1 (x = \log(S) = 0). The parameters associated with the stochastic processes of the variables can be found in Choi and Marcozzi (2001). The values used are: T = 0.5, \Theta^d = \Theta^f = 0.005, \sigma_{xx} = 0.0918, \sigma_{xdl} = \sigma_{xdr} = 0.0032, \sigma_{dd} = \sigma_{ff} = 0.0026, and \sigma_{df} = 0.0005. The exact European standard call option value is 0.0835, in which monitoring occurs only once (column 2).
References


Goldman, B. M. H. B. Sosin and M. A. Gatto, 1979, "Path dependent options: "Buy at the low, sell at the high", *Journal of Finance*, 34, 1111-1127


Figure 1: Finite-difference grid at fixed time for a floating strike lookback option
Figure 2: Finite-difference grid at fixed time for a fixed strike lookback option
Figure 3: Floating Strike Lookback Call Option
Figure 4: Fixed Strike lookback Call Option
Figure 5: Trinomial tree