“Marking-to-Market’’ and Treasury-Bill Futures Prices: Some Empirical Evidence

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Abstract

Financial economists have not found empirical evidence of a “marking-to-market” effect in Treasury-bill futures contracts, despite a firm theoretical basis for its existence. Therefore, we speculate that confounding effects, possibly due to liquidity preferences, influence futures-forward price spreads. By using an empirical specification that allows for both effects, we present empirical evidence that Treasury-bill futures-forward price spreads are sensitive to the volatility of the underlying commodity in ways predicted by the theory of the marking-to-market effect.

Keywords: marking to market, futures pricing, forward prices, interest sensitive instruments

JEL Classification: G13

1. Introduction

Financial economists are puzzled by the lack of empirical support for the “marking-to-market” effect in the pricing of futures contracts. This is the effect, described in Cox, Ingersoll, and Ross (1981), that daily settlement should have on
the price of futures contracts relative to forward contracts, which lack this feature. That is, if the value of the underlying asset is negatively correlated with interest rates, then its futures price should be less than its corresponding forward price. This is because the conditions under which the holder of a long futures position receives early payments (when spot and futures prices rise) are those in which reinvestment of those payments is least attractive (interest rates fall). Conversely, holders of long positions must make early payments under precisely those conditions when the opportunity cost of cash increases.

These observations lead us to predict that futures prices should be less than forward prices for contracts on negatively interest-sensitive underlying assets. However, current empirical studies have not found the evidence predicted. Instead, they find that spreads between forward and futures prices are either economically insignificant (Rendleman and Carabini, 1979) or carry the wrong sign (Elton, Gruber, and Rentzler, 1984). Flesaker (1993) and Musiela, Turnbull, and Wakeman (1993) address the magnitude of the spread in a theoretical framework and predict that it will be small, at least for short maturities.

Figure 1 uses the data in this study to illustrate the futures and forward price spreads. The figure uses data for the 3-month T-bill contract from 1994 and 1995 to show futures-forward spreads as a function of days until expiration. The spreads are averages over all occurrences of contracts that have that number of days to expiration during the sample period.

Fried (1994), using earlier data, finds a very similar pattern with even larger spreads. For all expiration periods, futures prices tend to exceed forward prices, which is completely opposite to the marking-to-market effect. Fried proposes an explanation of this phenomenon, which he bases on transaction costs and liquidity preference in bond markets. He notes that because there is no forward market for bonds, empirical research uses implied forward rates to calculate futures-forward price spreads. He models a world that contains perfectly liquid short-term bonds and completely illiquid long-term bonds. In his model the investor preference for liquidity adds a premium to long-term bond returns. When he uses these bonds to compute implied forward rates, the liquidity premium in long-term bonds produces (implied) forward prices lower than those that would emerge on an explicit forward market in which transaction costs (and consequently the liquidity premium) is absent.

We refer to the resulting positive spread as the "liquidity effect," and note that this effect induces a difference between implied forward prices of the type used in empirical research and the theoretical construct of a forward price that would emerge in an explicit forward market with low or no transaction costs. Equation (1) summarizes the liquidity effect:

\[ \text{(1)} \]

\[ \text{The economic significance of this difference is a separate issue (Capozza and Cornell, 1979). Elton, Gruber, and Rentzler (1984) find that apparently significant arbitrage opportunities appear during their sample period.} \]
Figure 1

The futures and forward price spread

This figure shows the difference between interest rates based on futures prices and implied forward rates as a function of days until expiration. Futures prices are Chicago Mercantile Exchange contracts for delivery of 13-week Treasury bills. These futures quotes are adjusted to be consistent with reported T-bill cash prices. Implied forward prices are calculated from spot prices for bills of the appropriate maturity as reported in The Wall Street Journal. The spreads are averages over all occurrences of contracts that have that number of days to expiration based on daily observations for the period January 2, 1994–December 6, 1995.

\[(\text{implied}) \text{ forward price} < (\text{explicit}) \text{ forward price}\] (1)

Fried (1994) looks for evidence of a liquidity effect by using observed futures prices as a proxy for the (empirically nonexistent) explicit forward contract. The positive futures to (implied) forward spread noted above is consistent with his theory. Moreover, he finds that variations in the future-forward spread can be explained by variables that might be correlated with investor demand for liquidity. He interprets these results as supporting the idea that a liquidity effect influences forward-futures spreads.

Fried (1994) is not interested in the issue of marking to market, although he acknowledges its existence in a footnote. He argues that finding a positive futures-
forward spread despite the negative spread predicted by the marking-to-market effect constitutes even stronger evidence of the liquidity effect he documents. Perhaps to be consistent with his empirical work, Fried refers to the explicit contracts in his theoretical model as “futures” contracts even though they contain no provision for marking to market and require no payments until expiration. Thus, strictly speaking, they are forward contracts, rather than futures contracts.

As noted above, theoretical studies of the marking-to-market effect (Cox, Ingersoll, and Ross, 1981) indicate that in a world free of transaction costs, marking to market implies futures prices lower than forward prices. Because the forward prices of these models carry no transaction costs, they correspond best to what are called “explicit” forward prices in Equation (1). In the language of Equation (1), the marking-to-market effect induces a spread between futures prices, which are tolerably approximated by observed futures prices, and the construct of a transaction-cost-free explicit forward price:

\[
(\text{explicit}) \text{ futures price} < (\text{explicit}) \text{ forward price}
\] (2)

We can not study the spread in either equation directly, because there are no explicit forward markets. This paper offers empirical support for the presence of a marking-to-market effect by demonstrating that the observed futures to (implied) forward spread is related to variables that should influence the magnitude of the marking-to-market effect. Section 2 outlines the implications of these two branches of the literature for the specification of the futures-forward spread. Section 3 describes the data, Section 4 presents the results, and Section 5 presents the conclusion.

2. Model specification

The theory of the marking-to-market effect is developed in the literature that discusses stochastic modeling of the term structure. We follow the structure proposed by Heath, Jarrow, and Morton (1992), who treat forward rates as the basic element by which the driving stochastic process is defined. In this structure, if \( f(t,T) \) is the instantaneous forward rate at time \( t \) for time \( T \), Heath, Jarrow, and Morton (1992) assume that forward rates follow a diffusion process in the form

\[
df(t,T) = u(t,T)dt + \sigma(t,T)dW(t)
\] (3)

where \( u(t,T) \) and \( \sigma(t,T) \) are drift and volatility parameters that can depend on the level of the term structure itself, and \( dW(t) \) is a scalar standard Wiener process used to model the single source of uncertainty. Heath, Jarrow, and Morton (1992) show that, given initial forward rates and a structure for the volatilities of all forward rates, interest rate claims can be uniquely priced without explicitly modeling the market price of risk. Using the forward rate process of Equation (3) and assuming continuous marking to market, Jarrow (1988) and Musiela, Turnbull, and Wakeman (1993) derive expressions for forward and futures prices and their spreads. To
implement these expressions empirically, we consider the special case in which the
volatility of the forward rate process is constant over time and is the same for all
forward rates, i.e., \( \sigma(t, T) = \sigma \). In this case, the Heath, Jarrow, and Morton (1992)
model is equivalent to the Ho and Lee (1986) term structure model, with the added
assumption of constant volatility. Since the Ho and Lee model treats spot (rather
than forward) rates as the element by which the underlying stochastic process is
defined, this implies that in the context of our empirical study, we can interpret
volatility as the volatility of either the spot or the forward rates.

An additional advantage of the constant volatility assumption is that it eliminates
the path dependency of the interest rate process that generally characterizes the
Heath, Jarrow, and Morton (1992) model. Constant volatility models satisfy the
conditions set forth in Hull and White (1993) and Ritchken and Sankarasubramanian
(1995) under which Heath, Jarrow, and Morton (1992) processes have a Markov
structure.

In our model forward and futures contracts expire at time \( T \) and are written
on a pure discount bond maturing at time \( T' \) with face value of \$1.00. Thus \( T < T' \).
Under the constant volatility assumption, the expression for the log difference
between the prices of a futures contract and a forward contract (Musiela, Turnbull,
and Wakeman, 1993; Flesaker, 1993) becomes:

\[
\lambda_t = - \frac{1}{2} \sigma^2 (T-t) (T'-T)
\]  

where \( \lambda_t \) is log futures forward spread; \( t \) is the current date, so that \( (T-t) \) is the time
remaining until expiration; and \( (T'-T) \) is the maturity of the underlying bond at the
time of delivery. Assuming constant volatility for purposes of this derivation means
market participants behave as if they expected market volatility to remain unchanged
throughout the life of the contract.

Since \( T'-T \) and \( \sigma^2 \) are positive, our model indicates that this component of the
spread is less than zero, and reflects the reduction in value caused by the marking-
to-market effect on the futures price. We also note that because our data come from
the market for 91-day Treasury bills, in our sample the term \( (T'-T) \) is a constant
(i.e., 91 days). Thus, this term can be absorbed in a constant. In addition, we
normalize by dividing the equation by \( (T-t)^2 \), making the left-hand side a time-
adjusted spread. With these changes, Equation (4) becomes:

\[
\lambda_t^* = \lambda_t / (T-t)^2 = \alpha_t \sigma^2
\]  

where:

\( \lambda_t^* \) = the time-adjusted log futures forward spread,
\( \alpha_t \) = is a constant, and
\( \sigma^2 \) = is the instantaneous variance of the diffusion process governing forward interest rates.
As indicated above, the marking-to-market effect requires that $\lambda^*_t$ be negative. Furthermore, the absolute value of this effect increases with volatility, that is, increasing volatility reduces (i.e., makes more negative) the adjusted spread.

Fried's (1994) theory of the liquidity component is less formally explicit than the term structure literature. His model allows bonds of only two maturities, long-term illiquid and short-term liquid bonds, which represent the two bonds used to construct the implicit forward contract. From these he derives the implication of a positive spread.

Fried (1994) also considers variables such as the difference between the Fed funds rate and the T-bill rate, the ratio of personal income to the money supply, the ratio of near-maturity government securities to money, etc., as measures of the demand for liquidity.

Since our study uses daily observations over a relatively short (two-year) period, such macroeconomic variables are unlikely to have noticeable explanatory power. For this reason, we treat the liquidity effect as constant, and allow for it by adding a constant to Equation (5). With this addition, the basic equation becomes:

$$\lambda^*_t = \alpha_0 + \alpha_1 \sigma^2$$

(6)

The marking-to-market effect predicts that $\alpha_1$ will be negative. Nevertheless, a positive $\alpha_0$ will make the net spread positive. This result is predicted by the liquidity effect and is consistent with empirical evidence.

3. Data

In our study we use the prices of T-bill futures contracts traded on the International Monetary Market of the Chicago Mercantile Exchange (CME). The assets underlying these contracts are three-month (13-week) U.S. Treasury bills. The T-bill futures contract has a March, June, September, and December (MJSD) delivery cycle with the last trading day being the business day before the U.S. government issue of 13-week Treasury bills. We use intraday transaction data for T-bill futures contracts of MJSD of 1994 and 1995, traded from January 2, 1994-December 6, 1995.

For each spread observation we use a futures price and a corresponding (implicit) forward price. The implicit forward price comparable to a futures contract with delivery in $d$ days is constructed from spot prices for bills maturing in $d$ and $d+91$ days. We collect spot price data from the daily Treasury-bill rates reported in The Wall Street Journal. Limitations on the maturities of traded Treasury bills confine these observations to the two nearest maturity futures contracts. Therefore, we choose for each day the first two futures contracts and four spot Treasury bills that are necessary to compute matching forward prices. We show the trading and maturity data in Table 1.

Our data matching procedure produces a total of 687 observations. We adjust both the implied forward prices and futures prices to correspond to the invoice
Table 1
Construction of futures-forward spreads
This table presents a list of the final trading date of each futures contract used in the sample along with the maturity date of the associated T-bill.

<table>
<thead>
<tr>
<th>Contracts</th>
<th>Last Trading Date</th>
<th>Underlying T-bill Maturity</th>
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</thead>
<tbody>
<tr>
<td>94 March</td>
<td>03/02/94</td>
<td>06/02/94</td>
</tr>
<tr>
<td>94 June</td>
<td>06/22/94</td>
<td>09/22/94</td>
</tr>
<tr>
<td>94 Sep</td>
<td>09/14/94</td>
<td>12/15/94</td>
</tr>
<tr>
<td>94 Dec</td>
<td>12/07/94</td>
<td>03/09/95</td>
</tr>
<tr>
<td>95 Mar</td>
<td>03/01/95</td>
<td>06/01/95</td>
</tr>
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<td>95 June</td>
<td>06/21/95</td>
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<td>09/13/95</td>
<td>12/14/95</td>
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<tr>
<td>95 Dec</td>
<td>12/06/95</td>
<td>03/07/96</td>
</tr>
</tbody>
</table>

amount the exchange uses to compute required payment (per 100) when delivery occurs. This means adjusting from the annualized yields quoted to yields for a 90-day period that matches the maturity of the deliverable bills.

Because Chicago futures markets close at 2 p.m. Central Time (3 p.m. Eastern), almost one hour earlier than New York spot markets (Elton, Gruber, and Rentzler, 1984), the spreads could be subject to nonsynchronous trading problems. To see if this is the case, we calculate a measure of the spread, using the average of the closing futures price and the next day opening price. As Table 2 shows, the statistical properties of the two series are almost the same. All subsequent empirical analyses were also not significantly affected by changing the measure of the spread. We base our reported results on the spread between log of closing futures price and forward price.

To implement Equation (6) we estimate $\sigma^2$, the volatility of the forward rate. We obtain our estimates by using intraday transaction data from the futures market. From Equation (3), the variance of the instantaneous change in forward rates is:

$$\text{var}_t(df(t,T)) = \sigma^2(t,T)dt$$

(7)

However, the variance of a discrete change in the forward rate is:

$$\text{var}_t(\Delta ft(t,T)) = \sigma^2(t,T)\Delta t + o(\Delta t)$$

(8)

where $o(\Delta t) \to 0$ as $\Delta t \to 0$. Thus, given intraday transaction data for futures rates, we compute the differences in futures rates divided by the square root of the time between transactions. We use these differences for our estimate of the standard
deviation. This is the proxy for \( \sigma \) used in this study. As we noted earlier, when \( \sigma \) is assumed to be constant, it can be viewed as the volatility of the spot rate. In our sample, the number of transactions per day ranges from 16 to 142. Because the daily sample standard deviations can be statistically poor estimates when there are few transactions, we use the two-day moving average of daily standard deviations, weighted according to the number of transactions each day, as a proxy for \( \sigma \). (We also tried three-day and four-day moving average figures, but the empirical results are robust to the choice of moving averages.) Its square is used as the independent variable in our implementation of Equation (6).

Table 2 presents descriptive statistics for the data. These are also displayed according to time until delivery of the futures contract. As noted by Bliss and Ritchken (1996), volatility structures of the generalized Vasicek class, which includes the Ho and Lee (1986) specification employed here, generate forward rate volatilities that exponentially decline in maturity. In our sample, volatility tends to increase with maturity, consistent with Amin and Morton (1994).

4. Estimation and results

When we run a basic regression on Equation (6) with an additive error term, it produces highly serially correlated residuals. To avoid this problem, the reported specification includes a lagged value of the spread. Thus, the basic regression specification is:

\[
\lambda_i^* = \alpha_0 + \alpha_1 \sigma_i^2 + \rho \lambda_{i-1}^* + e_i
\]  

(9)

Table 3, Model 1, reports the results of the basic regression. The standard errors in parentheses are heteroskedasticity-consistent standard errors calculated according to White’s (1980) procedure. In this form, the coefficient \( \alpha_1 \) is negative and significant; therefore the results are consistent with a marking-to-market effect.

A regression such as Equation (6) could result in a spurious relation if the series involved were integrated of order 1, I(1). To test against this possibility, we performed Dickey/Fuller tests on the series for volatility (\( \sigma^2 \)) and the time-adjusted spread (\( \lambda^* \)). The results indicated rejection of non-stationarity. The fact that one series is an interest rate spread (rather than the level of interest rates) may make this rejection less surprising.

We also try alternative specifications, as shown in the Table 3. In equity markets, the volume of trade is related to the flow of information, which, in turn,
Descriptive statistics

The mean and standard deviation for future-forward price spreads, number of trades per day, and the estimated intraday variance of changes in futures rates based on daily observations for the period January 2, 1994—December 6, 1995. Futures prices are Chicago Mercantile Exchange contracts for delivery of 13-week Treasury bills. Implied forward prices are calculated from spot prices for bills of the appropriate maturity as reported in The Wall Street Journal. In the table, FU equals futures price (adjusted to be comparable to cash T-bill prices) and FO equals the forward price. \( \lambda \) is the \( \frac{\ln(\text{futures price})}{\ln(\text{forward price})} \), \( \lambda^* \) equals \( \frac{\ln(\text{futures price}) + \ln(\text{opening futures price}_{t-1})}{2} \), and \( \lambda^* \) is the \( \frac{\lambda}{(\text{time remaining until expiration})^2} \). \( \sigma^2 \) equals the estimated intraday variance of changes in futures rates. \( NT \) is the number of daily transactions. \( \rho_{NT\sigma^2} \) equals the correlation between \( \ln(NT) \) and \( \sigma^2 \). The values of \( \lambda \), \( \lambda^* \), and \( \sigma^2 \) are multiplied by one thousand.

| Days to Delivery | Number of Observations | FU mean | FU std | FO mean | FO std | FU - FO mean | FU - FO std | \( \lambda \) mean | \( \lambda \) std | \( \lambda^* \) mean | \( \lambda^* \) std | NT mean | NT std | \( \sigma^2 \) mean | \( \sigma^2 \) std | \( \rho_{NT\sigma^2} \) |
|------------------|------------------------|---------|--------|---------|--------|--------------|------------|------------------|-----------|------------------|------------|---------|----------|--------|----------|
| all              | 687                    | 98.71   | 0.20   | 98.67   | 0.20   | 0.04         | 0.02       | 0.41             | 0.21      | 0.41             | 0.21       | 11.55   | 23.11    | 18.69  | 11.32    | 0.28     | 0.47     | 0.14     |
| <30 days         | 131                    | 98.75   | 0.20   | 98.74   | 0.20   | 0.02         | 0.01       | 0.17             | 0.15      | 0.18             | 0.15       | 3.61    | 4.41     | 13.80  | 9.38     | 0.23     | 0.79     | 0.13     |
| 30–90 days       | 291                    | 98.71   | 0.21   | 98.67   | 0.21   | 0.04         | 0.02       | 0.42             | 0.19      | 0.42             | 0.19       | 8.71    | 6.19     | 20.05  | 11.35    | 0.25     | 0.32     | 0.16     |
| >90 days         | 265                    | 98.69   | 0.18   | 98.64   | 0.18   | 0.05         | 0.02       | 0.51             | 0.16      | 0.51             | 0.16       | 2.56    | 1.10     | 19.61  | 11.57    | 0.35     | 0.39     | 0.15     |
Table 3

Regression results

Regression of time adjusted futures-forward spread on futures volatility, number of transactions, and days until delivery. Data are daily observations for the period January 2, 1994–December 6, 1995. Futures prices are Chicago Mercantile Exchange contracts for delivery of 13-week Treasury bills. Implied forward prices are calculated from spot prices for bills of the appropriate maturity as reported in The Wall Street Journal. \( \lambda_s \) equals \( \frac{\ln(\text{futures price}) - \ln(\text{forward price})}{(T-t)^2} \). NT equals the number of daily transactions. \( T-t \) equals the number of days until delivery. \( D_1 \) equals 1 if \( T-t < 30 \) days, otherwise 0. \( D_2 \) equals 1 if \( 30 < (T-t) < 90 \) days, otherwise 0. \( D_3 \) equals 1 if \( (T-t) > 90 \) days, otherwise 0. \( \sigma^2 \) equals the estimated intraday variance of changes in futures rates. Heteroskedasticity-consistent standard errors of White (1980) are in parentheses.

\[
\lambda_s = \alpha_0 + \alpha_1 \sigma^2 + \beta_1 \ln(NT) + \gamma_1 D_1(T-t) + \gamma_2 D_2(T-t) + \gamma_3 D_3(T-t) + \rho \lambda_{s,t} + \epsilon_t
\]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>0.004 (0.001)</td>
<td>0.002 (0.004)</td>
<td>0.006 (0.005)</td>
<td>0.002 (0.006)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>-8.538 (3.270)</td>
<td>-8.641 (3.326)</td>
<td>-8.122 (3.424)</td>
<td>-8.339 (3.539)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.0005 (0.001)</td>
<td>0.0005 (0.001)</td>
<td>0.001 (0.001)</td>
<td>0.001 (0.001)</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>0.014 (0.038)</td>
<td>0.012 (0.019)</td>
<td>0.014 (0.017)</td>
<td>0.014 (0.017)</td>
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<tr>
<td>( \gamma_2 )</td>
<td>0.006 (0.010)</td>
<td>0.006 (0.010)</td>
<td>0.006 (0.010)</td>
<td>0.006 (0.010)</td>
</tr>
<tr>
<td>( \gamma_3 )</td>
<td>0.917 (0.065)</td>
<td>0.918 (0.064)</td>
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<td>0.761</td>
<td>0.762</td>
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</table>


To avoid the possibility that this represents an important component of volatility that might be excluded from our proxy, \( \sigma^2 \), the specification for Model 2 also includes the number of trades per day. Table 1 shows that the number of trades is consistent for delivery dates beyond 30 days. We also note that the correlation of this variable with \( \sigma^2 \) is low, so that using both variables in the same regression is unlikely to cause multicollinearity problems. The positive coefficient estimate for this variable is inconsistent with the hypothesis that it adds information about volatility. Its inclusion does not appreciably change the estimated coefficient of \( \sigma^2 \).

In the specifications of Models 1 and 2, we use time until expiration of the futures or forward contract only to calculate the time adjusted spread as the theory of the marking-to-market effect requires. To check against the possibility that this
formulation misspecifies the role of time because of time-dependent liquidity effects, or for any other reason, Model 3 includes indicator variables for three separate time intervals: one for contracts “near” delivery (delivery in 30 days or less), one for contracts requiring delivery in the “intermediate” term (31 to 90 days), and one for “distant” delivery (more than 90 days). None of these indicator variables prove to be significant, nor do they have much affect on the estimates of the remaining coefficients. Model 4, which includes both time indicator variables and number of trades, also changes little.3

Finally, we further check the possibility that the time-adjusted spread variable might not fully and correctly capture the influence of time to expiration. Table 4 shows regression results for specifications identical to those of Table 3 except we replace the single variable $\sigma^2$ with interaction terms between $\sigma^2$ and the time indicator variables. In the basic specification (Table 4, Model 1) the volatility coefficients are all negative and significant at the 0.05 significant level. The volatility coefficient estimate for observations with less than 30 days to expiration is relatively larger than are those with more than 30 days to expiration. However, when we include other explanatory variables, such as the number of trades per day and the time-to-expiration dummies, (Table 4, Models 2-4), the volatility coefficients for observations with more than thirty days to expiration become less significant or insignificant.

5. Conclusions

Although the literature provides ample reasons to expect a marking-to-market effect in futures-forward price spreads, financial economists have generally been unable to find empirical evidence of this effect by examining the spreads themselves. This has led to speculation that confounding effects, possibly due to liquidity preferences, could also influence these spreads. By using an empirical specification that incorporates both effects, our paper presents evidence that futures-forward price spreads are, indeed, sensitive to the volatility of the underlying commodity in ways predicted by the theory of the marking-to-market effect. Although much remains to be learned about futures-forward price spreads, the results presented here suggest that the marking-to-market effect is one component of their behavior.

3 It could be argued that agents take account of more than recent historical information in forming their view of the volatility of the interest rate process to incorporate in the futures-forward spread. One possible approach to capturing this additional information is to consider an alternative instrument whose price depends on the volatility of the interest rate process. We investigate this possibility using implied standard deviations calculated from options on T-bill futures. We check for the possibility that this variable contains additional valuable information, by adding the option implied volatilities to the regression in Model 4. The option implied volatilities were obtained from Bridge, Inc. In this regression, the coefficient of implied volatility is not statistically significant, although it does have the appropriate negative sign. The estimated coefficient of historical volatility remains negative and significant.
Table 4

Regression results

Regression of time adjusted futures-forward spread on futures volatility, number of transactions, and days until delivery. Specification is as in Table 3 except the effect of volatility is permitted to differ in each time interval. Data are daily observations for the period January 2, 1994–December 6, 1995. Futures prices are Chicago Mercantile Exchange contracts for delivery of 13-week Treasury bills. Implied forward prices are calculated from spot prices for bills of the appropriate maturity as reported in The Wall Street Journal. \( \lambda^* \) equals the \( \ln(\text{futures price}) - \ln(\text{forward price}) \)\((T-t)^2 \). NT is the number of daily transactions. \( T-t \) equals days until delivery. \( D_1 \) equals 1 if \( T-t < 30 \) days, otherwise 0. \( D_2 \) equals 1 if \( 30 < T-t < 90 \) days, otherwise 0. \( D_3 \) equals 1 if \( T-t > 90 \) days, otherwise 0. \( \sigma^2 \) equals the estimated intraday variance of changes in futures rates. Heteroskedasticity-consistent standard errors of White (1980) are in parentheses.

\[
\lambda^* = \alpha_0 + \alpha_1 D_1 \sigma_1^2 + \alpha_2 D_2 \sigma_2^2 + \alpha_3 D_3 \sigma_3^2 + \beta_1 \ln(NT) + \gamma_1 D_1(T-t) + \gamma_2 D_2(T-t) + \gamma_3 D_3(T-t) + \rho \lambda^* + \epsilon
\]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
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<td>( \alpha_0 )</td>
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References


