Practice Problems and Assignment #2  
PHIL 422  
Dove/Woodbridge

Practice Problems  
[Note: any problem involving quantifiers or identity will be for after Thursday’s lecture.]

I. Logical Truth. Show that each of the following is logically true.

1. \((F(a,b) \rightarrow \neg (G(d,e,c) \land \neg F(a,b)))\)
2. \((A(a) \lor \neg (B(c) \land \neg A(a)))\)
3. \(((((A(a) \land B(b)) \lor (A(a) \land \neg B(b))) \lor (\neg A(a) \land B(b))) \lor (\neg A(a) \land \neg B(b)))\)
4. \((F(b) \land (a = b)) \rightarrow F(a)\)
5. \((\forall x F(x) \lor \exists y \neg F(y))\)

II. Logically False. Show that each of the following is logically false.

1. \(\neg (F(a,b) \rightarrow (G(a,b) \lor F(a,b)))\)
2. \((F(a) \land \neg F(b)) \land (a = b)\)
3. \((\forall x (F(x) \rightarrow G(x)) \land \exists y (F(y) \land \neg G(y)))\)

III. Entailment. Show that the following entailments hold.

1. \((F(a) \land F(b)), \neg F(b) \models G(a)\)
2. \((F(a,b) \lor G(c,d), (F(a,b) \rightarrow A(a)), (G(c,d) \rightarrow B(b)) \models (A(a) \lor B(b))\)
3. \((C(a,b) \lor C(b,c)), (b = c), \neg C(a,c) \models C(b,c)\)
4. \((\forall x (F(x) \rightarrow G(x)), \forall y (G(y) \rightarrow H(y)), \forall z (\neg J(z) \rightarrow \neg H(z)) \models \forall w (F(w) \rightarrow J(w))\)

IV. Inconsistency. Show that the following are inconsistent.

1. \(\{ (A(a) \lor A(b)), (A(a) \rightarrow G(c)), (G(c) \rightarrow \neg A(b)), (A(b) \rightarrow G(c)), (G(c) \rightarrow \neg A(a)) \}\)
2. \(\{ (A(a) \lor A(b)), (A(a) \rightarrow C(d)), (A(b) \rightarrow C(d)), \neg (C(d) \lor C(e)) \}\)
3. \(\{ (a = b), (b = c), (A(a) \land \neg A(c)) \}\)
4. \(\{ \forall x (F(x) \rightarrow G(x)), \exists y (F(y) \land H(y)), \neg \exists z G(z) \}\)

Assignment #2 (Due Tuesday, October 2 at the beginning of class.)

Show that the following entailment holds. [5 points]

1. \(\forall x (F(x) \rightarrow G(x)), \forall y (G(y) \rightarrow H(y)), \forall z (\neg J(z) \rightarrow \neg H(z)) \models (\exists x F(x) \rightarrow \exists w J(w))\)

Show that the following is logically false. [5 points]

2. \(\exists x (F(x) \land G(x)) \land (\neg \exists y F(y) \land \exists z G(z))\)