THIRD TEST

(OPEN BOOK/NOTES. YOU HAVE TWO HOURS TO COMPLETE THIS TEST)

1. DEFINING RULE CORRECTNESS: (15 points). Explain why we need to re-define the notion of the upward correctness of a Tree Method rule as it is stated in D16’, once we add the relevant rules to the Tree Method to extend it to Predicate Logic. Be as specific as you can about which rule(s) from the Tree Method this modification in the definition pertains to and why it is necessary for keeping all the rules upwardly correct.

2. PROVING RULE CORRECTNESS: (25 points). Prove the downward correctness (according to the modified definition D15’ of downward correctness) of the ∀ Rule from the Tree Method. Remember that this proof involves two cases, pertaining to two different situations with names. Make this proof as complete and precise as you can.

3. DOWNWARD ADEQUACY: (20 points). Explain what is meant by the Downward Adequacy of our deductive apparatus, the Tree Method. Then, identify which meta-logical feature this property is equivalent to and prove it is equivalent. [NOTE: This involves both restating the meta-logical feature in terms relevant to our system and proving that Downward Adequacy is equivalent to that restatement. It will help in doing this proof to express Downward Adequacy in formal/symbolic shorthand.]

4. MATHEMATICAL INDUCTION FOR NUMBERS: (20 points). Using strong mathematical induction prove that every polygon with n+2 sides (where n is any positive integer) has angles that sum to a total of (n x 180) degrees. [HINT: this isn’t really about the set of polygons—at least, that isn’t the induction set. You may appeal to the geometric facts that the angles of any triangle sum to 180 degrees and that adding one additional side to any polygon to form a new polygon is equivalent to tacking a triangle onto one of its sides. In the Induction Step, make your arbitrary level k + 1. You will then basically do weak induction in the context of doing strong induction. Explain when this happens.]

5. MATHEMATICAL INDUCTION FOR LOGIC: (20 points). Using strong mathematical induction, prove that a formal language for Sentential/Propositional Logic whose only sentential connectives are the material conditional and disjunction (“Iffory Logic”) cannot express any contradictions.

EXTRA CREDIT: (5 points). Explain the difference between strong mathematical induction and weak mathematical induction. Then explain why it is that we can use weak mathematical induction when we use the technique to determine facts about the numbers, but we must use strong mathematical induction when we use the technique to determine facts about formal languages and formal systems.