Open Branch Interpretations
PHIL 422
Dove/Woodbridge

Now that you are experts at using trees, we want to add semantic considerations.

As review, remember the following.

1. To test whether \( \phi \) is a theorem, we test \( |\neg\phi| \), i.e., we run a tree for \( \Sigma = \{\neg\phi\} \)
2. To test whether \( \phi \) is an anti-theorem, we test \( |\neg\neg\phi| \), i.e., we run a tree for \( \Sigma = \{\phi\} \)
3. To test whether \( \phi \) is neutral we test both \( |\neg\phi| \) and \( |\neg\neg\phi| \).
4. To test whether \( \Gamma |\neg\phi \), we run a tree for \( \Sigma = \Gamma \cup \{\neg\phi\} \)
5. To test whether a set is compatible, we run a tree for the members of the set.
6. To test whether a pair of sentences, \( \phi \) and \( \psi \), are coupled, we test \( |\neg(\phi \leftrightarrow \psi)| \), i.e., we run a tree for \( \Sigma = \{\neg(\phi \leftrightarrow \psi)\} \)

In each test, the resulting tree is either open (i.e., it has at least one open branch) or closed (i.e., all of its branches are closed). If it is open, then there is at least one branch on the tree for which no sentence \( \phi \) and its negation, \( \neg\phi \) both occur. Each open branch will be considered separately. Remember, a literal is an atom or a negated atom.

Consider the tree for \( \Sigma = \{\forall x(F(x) \rightarrow G(x)), \exists x(F(x) \land G(x))\} \)

\[
\forall x(F(x) \rightarrow G(x)) \checkmark_a \\
\exists x(F(x) \land G(x)) \checkmark \\
| \\
F(a) \land G(a) \checkmark \\
| \\
F(a) \\
G(a) \\
| \\
F(a) \rightarrow G(a) \checkmark \\
/ \\ \ \ \ \\
\neg F(a) \and G(a) \times \\
\otimes \bullet
\]

The open branch contains the following sentences: “\( \forall x(F(x) \rightarrow G(x)) \),” “\( \exists x(F(x) \land G(x)) \),” “\( F(a) \land G(a) \),” “\( F(a) \),” “\( G(a) \),” and a second example of “\( G(a) \).” An open branch interpretation makes all of the sentences on the open branch true.

Here’s the Method:

**Step 0:** Identify ALL of the literals on the branch. [In practice, circle them.]

**Step 1:** Set each literal’s constant(s) on the branch equal to denote a number as follows:

“\( a \)” denotes 1, “\( b \)” denotes 2, “\( c \)” denotes 3, …
Step 1.5: For every freestanding identity sentence, (re-)set both the constants to denote the lower of the two numbers from step 1, and eliminate the higher number from consideration. So, if the identity sentence is “(a = b)”, then, after step 1 gives you that “a” denotes 1 and “b” denotes 2, now you must reset “b” to denote 1 and eliminate 2 from consideration.

Step 2: Add the number for each constant from every literal wff on the branch to the domain of the interpretation, D.

Step 3: Add the number for each constant from every atomic wff on the branch to the extension for the predicate (and ordered pairs to the extension of relations, and ordered triples to the extension of tri-lations, etc.) that the (sequence of) name(s) is(are) combined with in the atom wff.

Step 4: Check to make sure that, for each negated atom on the branch, the relevant items (the numbers for predicates, the ordered pairs of numbers for relations, the ordered triples of numbers for tri-lations, etc.) DO NOT OCCUR in the extension of the predicates.

Step 5: Test the resulting interpretation against Σ, i.e., plug the interpretation into the sentences that are in Σ to see whether it makes them all true.

That’s it.

What you’ve constructed is an Open Branch Interpretation, an OBI. [Pronounced “Oh-Bee”, as in “Obi-Wan Kenobi”.

So, consider the open tree constructed above. The literals on this branch are “F(a)” and “G(a)”. These sentences employ just one name, “a”, so set it to denote 1. That means the OBI is as follows. D = {1}, F: {1}, G: {1}. This interpretation makes “∀x(F(x) → G(x))” true and makes “∃x(F(x) ∧ G(x))” true, so it shows that the set is consistent as well as compatible.

To practice, use any of the practice problems or homework problems. If the tree is open, construct and OBI on (at least) one open branch.