MATHEMATICAL INDUCTION PRACTICE PROBLEMS

1. Using **strong mathematical induction**, as employed on the Mathematical Induction Handout, prove the following claim, “Every odd positive integer is such that the sum of it and all the odd positive integers before it, in the ordered sequence of odd positive integers, is equal to the square of the position number of the given odd positive integer in the sequence.” [Hint: you will end up doing weak induction in the context of doing strong induction. Explain when this happens.]

2. Using **strong mathematical induction**, as employed on the Mathematical Induction Handout, prove that every positive integer \( n \geq 5 \) is such that \( 2^n > n^2 \). The Induction Set and Basis are obvious here, so the work comes in proving the Induction Step. [Hint: you will again basically do weak induction in the context of doing strong induction. Explain when this happens.]

3. Using **strong mathematical induction**, as employed on the Mathematical Induction Handout, prove the following claim, “Every polygon with \( n+2 \) sides (where \( n \) is any positive integer) has angles that sum to a total of \( (n \times 180) \) degrees.” [Hint: while this appears to be about polygons, the induction set is still the positive integers. You may appeal to the geometric facts that the angles of any triangle sum to 180 degrees and that adding one additional side to any polygon to form a new polygon is equivalent to tacking a triangle onto one of its sides. You will then do weak induction in the context of doing strong induction. Explain when this happens.]

4. Using **strong mathematical induction**, as employed on the Mathematical Induction Handout, prove the following claim, “A formal language whose only sentential connectives are disjunction and conjunction (“Orandy Logic”—governed by the usual Formation Rules and truth-tables—cannot express any tautology or contradiction.” Recall that the induction set here consists of the sentences of a formal language (which is not the formal language of our system), call it SFOL*. What we want to show to hold for every case is that each is contingent (not necessary, but not impossible). [Hints: complexity is the best way to partition the Induction Set, and the Basis is the same as it was in our proof of PST for SFOL—and for the same reasons. In proving the Basis, remember that atomic sentences can be either true or false, depending on what interpretation you are considering, so they will never be true on every interpretation, nor false on every interpretation. Finally, in the Induction Step, there are only two ways a wff of SFOL* of arbitrary level of complexity \( n = k+1 \) can arise: as a conjunction or a disjunction.]
5. Using **strong mathematical induction**, as employed on the Mathematical Induction Handout, prove the following claim, “A formal language whose only sentential connectives are conditional and conjunction—governed by the usual Formation Rules and truth-tables—cannot express any contradiction.” Recall that the induction set here consists of the sentences of a formal language (which is **not** the formal language of our system), call it SFOL# (or “Ifandy Logic”). [Hints: complexity is again the best way to partition the Induction Set, and the Basis is the same as it is in the proof that every sentence of SFOL* is contingent, and the proof of the Basis is basically the same. Note that here too, in the Induction Step, there are only two ways a wff of SFOL# of arbitrary level of complexity n=k+1 can arise: as a conditional or a conjunction.]

6. Using **strong mathematical induction**, as employed on the Mathematical Induction Handout, prove that the number of sub-wffs of any wff of SFOL with n connectives is less than or equal to 2n+1. Take ‘Sub(φ)’ to stand for the set of sub-wffs in a wff φ, and ‘|Sub(φ)|’ to stand for the number of members in that set, i.e., the number of sub-wffs that occur in a wff φ. So what you want to prove is that if φ has n connectives, then |Sub(φ)| ≤ 2n+1. [Hint: Think in terms of breaking a wff down into sub-wffs in steps, based on what kind of wff you have at each step. What kind of wff you have is determined by the string’s main connective. Note that even though we have 5 connectives, there are really only two patterns of breaking wffs down.]