1. Prove that at any conference, the number of people who have shaken an odd number of hands (assuming all handshakes are normal) is even.
This will be a Proof by Induction on the Number of Handshakes.

Set up: Let the collection of conference attendees be $C$. Partition $C$ into two subsets, $E$ and $O$, such that $E$ is the collection of conference attendees who have shaken an even number of hands, and $O$ is the collection of conference attendees who have shaken an odd number of hands. Let $O^n$ give the partition of $C$ of people in $O$ at the level of $n$ handshakes. And let $|O^n|$ be the size of the extension of $O$ for a given number of handshakes, $n$, that have so far occurred at the conference. We will show that for any number of handshakes, $|O^n|$ is even, i.e. $= 2k$ for some $k$.

2. Show that for any wff of FOL, the number of atoms in the wff is the number of binary operators in the wff, plus one. (Show that for any $\phi$ [i.e., for $\phi$ of any complexity $n$], $|\phi|_a = |\phi|_b + 1$.) This will be a Proof by Induction on the Number of Operators in $\phi$.

Set up: Let $|\phi|_a = \#$ of atoms in $\phi$. Let $|\phi|_b = \#$ of binary operators in $\phi$. Let $|\phi|_o$ = the number of operators (including both binary and unary operators) in $\phi$. Let “@$” be any unary operator, i.e., “¬” or “∃” or “∀”. Let “*$” be any binary operator, i.e., “∧” or “∨” or “→” or “↔”.

3. Let Mr. T, $\mathcal{T}^x$, be an interpretation that makes every atomic sentence of FOL true. Show that Mr. T makes any wff $\phi$ true where $\phi$ does NOT contain any unary operators.

This will be a Proof by Mathematical Induction on $|\phi|_b$ (the Binary Complexity of $\phi$)