PHIL 114: Introduction to Symbolic Logic

Syntax: Formation Rules for Sentences of First-Order Logic (FOL)

0. (Atomic Sentence Rule) Any capital letter (with or without numerical subscripts, e.g., ‘A’, ‘B’, ‘A_1’, ‘B_1’, ‘C_5’, ‘D_9’, ‘E’, etc.) is an atomic sentence of FOL. Any n-place predicate (capital letter or capitalized “short word”) followed by n names (‘a’, ‘b’, ‘c’, ‘d’, ‘n_1’, ‘n_2’, ‘n_3’…), inside parentheses, separated by commas (except in the case of identity claims), is an atomic sentence of FOL (e.g., ‘F(a)’, ‘G(a, b)’, ‘M(c, d, n_3)’, ‘a = b’, etc.). **Extension to well-formed formula (or wff):** Any n-place predicate followed by n *individual terms*—either names or *variables* (e.g., ‘x’, ‘y’, ‘z’, ‘u’, ‘v’, ‘w’, ‘x_1’, ‘x_2’, ‘y_3’, etc.)—inside parentheses, separated by commas (except in the case of identity claims), is an atomic wff of FOL (e.g., ‘F(x)’, ‘G(a, y)’, ‘M(b, z_3, z_8)’, ‘a = x’, ‘y = z’, etc.).

1. (Negation Rule) If φ is a sentence/wff of FOL, then so is ¬φ (no parentheses added).

2. (Conjunction Rule) If φ and ψ are both sentences/wffs of FOL, then so is (φ ∧ ψ) (here, and in 3-5, parentheses are needed).

3. (Disjunction Rule) If φ and ψ are both sentences/wffs of FOL, then so is (φ ∨ ψ).

4. (Conditional Rule) If φ and ψ are both sentences/wffs of FOL, then so is (φ → ψ).

5. (Biconditional Rule) If φ and ψ are both sentences/wffs of FOL, then so is (φ ↔ ψ).

6. (Universal Quantifier Rule) If φ is a wff of FOL and υ is a variable, then ∀υφ is a wff of FOL, and any occurrence of υ in ∀υφ is said to be bound.

7. (Existential Quantifier Rule) If φ is a wff of FOL and υ is a variable, then ∃υφ is a wff of FOL, and any occurrence of υ in ∃υφ is said to be bound.

PC (Parentheses Convention) If after applying and re-applying the Formation Rules to build up to a sentence/wff of FOL, an application of the final rule you use places parentheses around the whole formula as the outer-most symbols, these outer-most parentheses can be dropped (or kept).

These Formation Rules determine what is and what is not a wff/sentence of FOL: **(Closure Rule)** A string of symbols counts as a wff of FOL if, and only if, it can be constructed via application and re-application of these rules. A sentence of FOL is a wff of FOL with no free (unbound) variables.