Formation Rules for Sentences of FOL (those we have so far)—Note a slight re-numbering

1. (Atomic Sentence Rule) Any capital letter (followed by any number of primes, e.g., A, B, A', C', D', etc.) is an atomic sentence of FOL. Any n-place predicate followed by n names (inside parentheses, separated by commas) is an atomic sentence of FOL (e.g., H(a) or G(a, b) or J(c, d, e), etc.).

2. (Negation Rule) If \( \phi \) is a sentence of FOL, then so is \( \neg \phi \) (no parentheses get added).

3. (Conjunction Rule) If \( \phi \) and \( \psi \) are both sentences of FOL, then so is \( (\phi \land \psi) \) (parentheses always added).

4. (Disjunction Rule) If \( \phi \) and \( \psi \) are both sentences of FOL, then so is \( (\phi \lor \psi) \) (parentheses always added).

PC (Parentheses Convention) If after applying and re-applying the Formation Rules to build up to a sentence, an application of the final rule you use places parentheses around the whole formula as the outer-most symbols, these outer-most parentheses can be dropped (or kept).

These Formation Rules determine what is and what is not a proper sentence of FOL. If a string of symbols can be constructed via application and re-application of these rules (more will be added later), then it counts as a sentence. If it cannot, then it is not a sentence of FOL.

Examples:

I) Consider the string: \( B \lor (\neg C \land \neg A) \). Is this a sentence of FOL? Here is a proper explanation. (Notice that new lines in the explanation will cite a Rule and the Lines that supply the string(s) to which the Rule applies. Cite only the immediately relevant Lines. In citing a Rule you can use either the rule number or the name.

1. A, B, and C are all sentences of FOL by Rule 1 (Atomic Sentence Rule).
2. \( \neg A \) and \( \neg C \) are sentences of FOL by Rule 2 (Negation Rule) and Line 1.
3. \( (\neg C \land \neg A) \) is a sentence of FOL by Rule 3 (Conjunction Rule) and Line 1.
4. \( (B \lor (\neg C \land \neg A)) \) is a sentence of FOL by Rule 4 (Disjunction Rule) and Lines 1, 3 (Note that the outer parentheses are present; they must be for Rule 4 to be properly applied).
5. \( B \lor (\neg C \land \neg A) \) is not a sentence of FOL because no Formation Rule will produce this from Line 4 (Rule 2 does not add parentheses—the string has an extra left one).

II) Consider the string: \( \neg((\neg A \land (C \lor B)) \). Is this a sentence of FOL?

1. A, B, and C are all sentences of FOL by Rule 1.
2. \( \neg A \) and \( \neg C \) are sentences of FOL by Rule 2 and Line 1.
3. \( (C \lor B) \) is a sentence of FOL by Rule 4 and Line 1.
4. \( (\neg A \land (C \lor B)) \) is a sentence of FOL by Rule 3 and Lines 2, 3.
5. \( \neg((\neg A \land (C \lor B)) \) is not a sentence of FOL because no Formation Rule will produce this from Line 4 (Rule 2 does not add parentheses—the string has an extra left one).
Syntax Exercises: Using the Formation Rules we have so far, determine whether the following strings of symbols are sentences of FOL. Give explanations like I did in the Examples.

1. \((\neg B \lor (\neg C \land A))\)
2. \(A \land (B \lor (\neg C))\)
3. \(F \lor (D \land (B \lor (\neg C \lor H)))\)
4. \(A \lor (\neg A' \land (A'' \lor (\neg A''' \land A''')))\)
5. \((\neg A \lor B) \land (C \land B)) \lor (B \lor (\neg C \land \neg A))\)