Some Guidelines for Constructing Shear Force and Bending Moment Diagrams

**SIGN CONVENTIONS**

+ $M$
+ $V$
- $M$
- $V$

**BOUNDARY CONDITIONS**

**Pinned LEFT End:**
- Reaction force, $A_x$ is unknown.
- Reaction force, $A_y$ is unknown.
- $V$ (shear force) = $A_y$ (reaction force)
- $M = 0$ (unless there is an applied moment at this point)
- Deflection, $y = 0$
- Slope, $\theta$ is unknown

**Pinned RIGHT End:**
- Reaction force, $B_x$ is unknown.
- Reaction force, $B_y$ is unknown.
- $V$ (shear force) = $-B_y$ (reaction force)
- $M = 0$ (unless there is an applied moment at this point)
- Deflection, $y = 0$
- Slope, $\theta$ is unknown

**Pinned LEFT End:**
- Reaction force, $A_x$ is unknown.
- Reaction force, $A_y$ is unknown.
- $V$ (shear force) = $A_y$ (reaction force)
- $M = 0$ (unless there is an applied moment at this point)
- Deflection, $y = 0$
- Slope, $\theta$ is unknown

**Roller Support at Right End:**
- Reaction force, $B_x = 0$.
- Reaction force, $B_y$ is unknown.
- $V$ (shear force) = $-B_y$ (reaction force)
- $M = 0$ (unless there is an applied moment at this point)
- Deflection, $y = 0$
- Slope, $\theta$ is unknown

**Clamped End:**
- Reaction force, $A_x$ is unknown.
- Reaction force, $A_y$ is unknown.
- Reaction moment, $M_A$ is unknown
- $V$ (internal shear force) = $A_y$ (reaction force)
- $M$ (internal bending moment) = $M_A$
- Deflection, $y = 0$
- Slope, $\theta = 0$

**Free End:**
- $V$ (shear force) = 0 (unless there is an applied point force at this end)
- $M = 0$ (unless there is an applied moment at this point)
- Deflection, $y$ is unknown
- Slope, $\theta$ is unknown
Example:
Draw the shear force and bending moment diagram for the following beam:

![Beam Diagram]

Step 1: Find the reaction forces at A and D.
Draw F.B.D.:
\[
\sum F_x = 0, \quad A_x = 0 \\
\sum M_A = 0, \quad (20 \text{ kips})(6 \text{ ft}) + (12 \text{ kips})(14 \text{ ft}) + (1.5 \text{ kips/ft})(8 \text{ ft})(28\text{ ft}) - (D_y)(24 \text{ ft}) = 0 \\
D_y = 26 \text{ kips} \\
\sum F_y = 0, \quad A_y + D_y - 20 \text{ kips} - 12 \text{ kips} - (1.5 \text{ kips/ft})(8 \text{ ft}) = 0, \quad A_y = 18 \text{ kips}
\]

Construction of the shear force diagram

![Shear Force Diagram]

- a) \( V = A_y = 18 \text{ kips} \) at point A.
- b) The shear force is constant until there is another load applied at B.
- c) The shear force decreases by 20 kips to \(-2 \text{ kips}\) at B because of the applied 20 kip force in the negative y direction.
- d) The shear force is constant until there is another load applied at C.
- e) The shear force decreases by 12 kips to \(-14 \text{ kips}\) at C because of the applied 12 kip force in the negative y direction.
- f) The shear force is constant until there is another load applied at D.
- g) The shear force increases by 26 kips to 12 kips at D because of the 26 kip reaction force in the positive y direction.
- h) The shear force decreases linearly from D to E because there is a constant applied load in the negative y-direction.
- i) The change in shear force from D to E is equal to the area under the load curve between D and E, \(-12 \text{ kips}\), \([A_{DE} = (-1.5 \text{ kips/ft})(8 \text{ ft}) = -12 \text{ kips}]\)
- j) The shear force at E = 0 as expected by inspection of the boundary conditions.
Construction of the Bending Moment diagram

\[
\begin{align*}
\text{V} & \quad 6 \text{ ft} \quad B \quad 8 \text{ ft} \quad C \quad 10 \text{ ft} \quad 8 \text{ ft} \quad E \\
18 \text{ kips} & \quad -2 \text{ kips} & \quad -14 \text{ kips} & \quad 12 \text{ kips} \\
\text{M} & \quad 108 \text{ kip-ft} & \quad 92 \text{ kip-ft} & \quad -48 \text{ kip-ft} \\
\end{align*}
\]

a) \( M = 0 \) at point A because it is a pinned end with no applied bending moment.
b) \( M_B = M_A + (\text{the area under the shear force diagram between A and B.}) \)
c) \( M_B = 0 + (18 \text{ kips})(6 \text{ ft}) = 108 \text{ kip-ft} \)
d) \( M_C = M_B + (\text{the area under the shear force diagram between B and C.}) \)
e) \( M_C = 108 \text{ kip-ft} - (2 \text{ kips})(8 \text{ ft}) = 92 \text{ kip-ft} \)
f) \( M_D = M_C + (\text{the area under the shear force diagram between C and D.}) \)
g) \( M_D = 92 \text{ kip-ft} - (14 \text{ kips})(10 \text{ ft}) = -48 \text{ kip-ft} \)
h) \( M_E = M_D + (\text{the area under the shear force diagram between D and E.}) \)
i) \( M_E = -48 \text{ kip-ft} + \frac{1}{2} (12 \text{ kips})(8 \text{ ft}) = 0 \text{ kip-ft (as expected)} \)