Needham, Chapter 5, Exercise 8, p. 258.
G. Palmer, January 1, 2016

8. Calculate, then draw on a picture, a possible location for $\log(1+i)$. Draw a small shape at $1 + i$. Use the amplitwist of $\log(z)$ to draw its image. Verify this using your computer.

Figure 1 was plotted prior to reading Vasco’s explanation of his plot. I subsequently added an explanation and dotted lines at $\ln \sqrt{2}$ and $i\pi/4$. We can see by inspection that the angle of $(1 + i)$ is $\pi/4$. We drew the blue vector $\vec{e}$ at $1+i$ with the same angle. It forms the first side of the square at $1+i$. The green vector at $1+i$ is $i\vec{e}$. The four sides of the square were plotted by multiplying $\vec{e}$ by successive powers of $i$ and adding the result to the endpoint of the previous side. The point at $\log(1+i)$ can be calculated with $\log(z) = \ln|z| + i\theta$ (p. 98).

$$\log(1+i) = \ln \sqrt{2} + i\pi/4$$

The amplitwist of $\log(1+i)$ is $1/(1+i)$ (p. 222). If we represent $(1+i)$ as $\sqrt{2} \ e^{i\pi/4}$, then the amplitwist is $$\frac{\sqrt{2}}{2} \ e^{-i\pi/4}$$. Since $\text{Arg}(\vec{e}) = \pi/4$ and the twist is $-\pi/4$, we can write

$$\text{Arg}(f(z) \vec{e}) = \text{Arg}(\frac{\sqrt{2}}{2} \ e^{-i\pi/4} \ r \vec{e}^{i\pi/4}) = 0,$$

which is the angle of the blue arrow in the smaller square based at $\log(1+i)$. From the dotted lines and corresponding colors, we can see that the right angles have been preserved, so $\log(z)$ is conformal.
Figure 1. Amplitwist of image at log(1+i)