26. Unlike inversion in a circle, show that Schwarzian reflection in an ellipse $E$ (see figure [31]) does not interchange the interior and the exterior. Indeed, how does $R_E(z)$ behave for large values of $|z|$?

We now see why on p. 256, Needham provided only the numeric form of $R_E$. He was leaving the symbolic derivation for this question, or the question has a numerical solution.

If $R$ exchanges the inside for the outside, then $R_E \circ R_E(z) = z$.

A single contradictory example is enough to show that an ellipse $E$ does not interchange the interior and the exterior. Is it the case with the example on p. 256?

$$R_E(z) = \frac{1}{3}(5z - 4\sqrt{z^2 - 3})$$

Try a point outside the ellipse, such as the point $2.5 + 1.6i$.

$$R_E(R_E(2.5 + 1.6i)) = 2.11 - 1.23i \quad (\text{See the Mathematica cell, below.})$$

We see that $R_E(R_E(2.5 + 1.6i)) \neq 2.5 + 1.6i$. Since the test for exchange of interior and exterior points fails in this case, we conclude that Schwarzian reflection in an ellipse $E$ does not in general interchange the interior and the exterior.

How does $R_E(z)$ behave for large values of $|z|$?

For large values of $|z|$, $R_E(z)$ falls outside the ellipse, so there can be no exchange between the inside and the outside. We can be sure of this when $|R_E(z)| = \frac{1}{3}(5z^2 - 4\sqrt{z^2 - 3}) > 2$, where 2 is the value of the largest semi-axis. We can see in particular that when $x$ is very large, where $z = x + iy$, $R_E(z)$ falls outside the ellipse when $|z| > 6$. 

```
Clear["Global`*"];

R[z_] := (1/3)(5Conjugate[z] - 4Sqrt[Conjugate[z]^2 - 3]);

R[R[2.5 + 1.6 I]]
```

Out[287]= $2.10878 - 1.22527i$