We plot the branch function $f_2(z)$ for $z$ starting at 0, where $f_2(0) = -1$ (Line 2 of Table 1). We choose some points that make a loop going anticlockwise around branch point 1. We then plot the points for $z$ and $f(z)$. Preliminary to writing the *Mathematica* program, we write pseudocode for $f_2(z)$, with the branch table data from line 2, columns 1 and 2, of Table 1:

$$\text{branchTable}[2] = \{\text{Pi}, 2\text{Pi}\}; \quad \{\text{increment to } \theta, \text{increment to } \phi\}$$

$$f_2(z) = \left\{ \begin{array}{l}
    r_1 = \text{Abs}[z-1], \quad r_2 = \text{Abs}[z-i] \\
    \theta = \text{Arg}[z-1] + 0, \quad \phi = \text{Arg}[z-i] + 2\text{Pi} \quad \text{(Advance } \theta \text{ by Pi)} \\
    \text{return } \sqrt{r_1} \sqrt{r_2} \ e^{(\theta/2 + \phi/3)}
\end{array} \right\}$$

```
 Clear["Global`*"]
 (* GLOBALS *)

(* PATHS OF Z IN Z PLANE; f(z) WILL BE TRAACKED *)

(* anticlockwise loop; conjugate to obtain clockwise loop *)
loop1 = \{0, .02 - .02 I, 0.8 - 0.2 I, 1.0 - 0.2 I, 1.2 - .1 I, 1.2 + .1 I, 1.0 + 0.2 I, 0.8 + 0.2 I, .02 + .02 I\};

(* circular loop *)
(* loop1 = ( Exp[I \#] +1) &/@Range[-Pi,Pi+.2,.2]; *)

(* branch table from Table 1, line 2, columns 1, 2; winds added to $\theta$ and $\phi$ in multifunctions *)
branchTable = \{(0, 0), (0, 2 Pi), (0, 4 Pi), (2 Pi, 0), (2 Pi, 2 Pi), (2 Pi, 4 Pi)\};
```
(* odds and ends *)
pr = 2.5; (* plot range *)
labelSz = 10;

r = 1.0; (* radius of circle in z-plane *)

(* COLORS *)
bgColor = White;
dotColor = Red;
curveColor = Red;

(* the six points for f(0) *)
f0CPts = {Exp[I Pi/3], -1, Exp[-I Pi/3], -Exp[I Pi/3], 1, -Exp[-I Pi/3]};

(* spc = " "; *) (* for inspection code *)

(* FUNCTIONS *)

v[z_] := {Re[z], Im[z]};
SetAttributes[{v}, Listable];

(* Inspection code
   Print["arg[z-1]: ", arg[z-1],
   " branchTable[[n,1]]: ", branchTable[[n,1]], " θ: ", θ, " φ: ", φ];
   (r1^î(1/2))( r2^î(1/3)) Exp[I (θ/2 + φ/3)]]; *)

(* BRANCH FUNCTIONS: BRANCHES 1-6 *)

(* n is index of multifunction by order of line in multifunction table; references global branchTable *)

(* Principal branch is fn[1, z] *)
(* This branch modified to accommodate Wolfram Arg[] branch cut from 0 to -∞. *)
(* One of the following functions could be dispensed with by adding a direction variable. *)

fn[n_, z_] := Module[{r1, r2, θ, φ},
   r1 = Abs[z-1.0]; r2 = Abs[z-I];
   θ = If[z == 0, Pi, 2 Pi + Arg[z-1]]; (* modification for branch cut *)
   ...
\[
\phi = \text{Arg}[z-I] + \text{branchTable}[[n, 2]];
\]
\[
(r1^\phi (1/2)) (r2^\phi (1/3)) \text{Exp}[I (\theta/2 + \phi/3)]
\]

fnCW[n_, z_] := Module[{r1, r2, \theta, \phi},
  r1 = Abs[z - 1.0]; r2 = Abs[z - I];
  \theta = If[z == 0, Pi, \text{Arg}[z - 1]]; (* modification for branch cut *)
  \phi = \text{Arg}[z - I] + \text{branchTable}[[n, 2]];
  (r1^\phi (1/2)) (r2^\phi (1/3)) \text{Exp}[I (\theta/2 + \phi/3)]
]

(* GRAPHICS PRIMITIVES *)

(* f(0) points *)

f0Pts =
  {\text{EdgeForm}[[\text{Thickness}[.003], \text{dotColor}]], \text{bgColor}, \text{Disk}[v \oplus \#, .030]} \& /@ f0Cpts;

zDots =
  {\text{EdgeForm}[[\text{Thickness}[.005], \text{dotColor}]], \text{bgColor}, \text{Disk}[v[\#], .035]} \& /\{0, 1, \text{I}\};

oDot = {\text{EdgeForm}[[\text{Thickness}[.005], \text{dotColor}]], \text{bgColor}, \text{Disk}[v[0], .035]};

(* replacement axes *)

xLn = \text{Line}[[{-pr, 0}, \{pr - .25, 0}]];
iLn = \text{Line}[[\{0, -pr\}, \{0, pr - .25\}]];
axes = {xLn, iLn};

(* CAPTIONS *)

cap0 = \text{Text}[\text{Style}["0", \text{labelSz}, \text{Bold}], v[0] - v[.15 + .15 I]];

zLabel = \text{Text}[\text{Style}["z plane", 16], \{1.5, -1.25\}];

fLabel = \text{Text}[\text{Style}["f(z) plane", 16], \{1.5, -1.25\}];

zPlaneCLabels =
  \text{Text}[\text{Style}[\text{ToString}[\{0, 1, \text{I}\}], \text{labelSz}, \text{Bold}], v[\#] - v[.15 + .15 I]] \& /\{0, 1, \text{I}\};

xLabel = \text{Text}[\text{Style}["x", \text{labelSz}, \text{Bold}], v[pr] - v[.15]]; iLabel = \text{Text}[\text{Style}["i", \text{labelSz}, \text{Bold}], v[pr I] - v[.15 I]];

axisLabels = {xLabel, iLabel};

(* GRAPHICS *)
Manipulate[
    loop = If[d == "anticlockwise",
        loop1[[1 ;; n]],
        Conjugate[loop1[[1 ;; n]]]]
    ];

GraphicsRow[
    {g1[
        Thick, curveColor, Line[v[loop]],
        Text[Style[d, Bold, curveColor], {1.5, -2}]
    ],
    If[d == "anticlockwise",
        g2[Thick, curveColor, Line[v[fn[2, H]] & @ loop]],
        g2[Thick, curveColor, Line[v[fnCW[2, H]] & @ loop]]
],
    Background -> bgColor, Spacings -> 0, ImageSize -> Medium],
Style["
    Principal branch \( f_1(z) = \sqrt{z - 1} \)
    
    Choose direction for \( f_2(z) \), where \( f_2(0) = -1 \), Bold, 8],
{{d, "anticlockwise", "loop"}, {"anticlockwise", "clockwise"}},
Style["z \, \text{random} \, 1 \, \text{from} \, \text{n}_2(0) = -1 \, \text{to} \, f_2(z) = 1", Bold, 8],
{n, 1, "points in curve"}, Range[1, Length[loop1], 1],
SaveDefinitions -> True