Unit 1 – Communication

Section A – Mathematical Notation

Base Objectives: Identify, interpret, and evaluate mathematical symbols and notation

The Four Basic Operators:

- **Addition** +
  - History: [all history notes taken from http://members.aol.com/jeff570/mathsym.html] Oresme (1323-1382) may have used a figure which looks like a plus symbol as an abbreviation for the Latin et (meaning "and") in *Algorismus proportionum*, believed to have been written between 1356 and 1361.

- **Subtraction** –

- **Multiplication** × ⋅ ∗ ()
  - History: In a manuscript found buried in the earth near the village of Bakhshali, India, and dating to the eighth, ninth, or tenth century, multiplication is normally indicated by placing numbers side-by-side (justaposition). × was used by Oughtred (1574-1660) in the *Key to Mathematics*, composed about 1628. The dot (⋅) was advocated by Leibniz (1646-1716). The asterisk (*) was used by Rahn (1622-1676) in 1659.

- **Division** ÷ /
  - History: The arrangement 8)24 was used by Stifel (1487-1567) in *Arithmetica integra*, which was completed in 1540. The colon (:) was used in 1633. The obelus (÷) was first used as a division symbol by Rahn (1622-1676) in 1659. In nineteenth century U. S. textbooks, long division is typically shown with the divisor, dividend, and quotient on the same line, separated by parentheses, as 36)116(3.
  - Students often have difficulty with the ordering of the numbers when dividing. Some of the hints similar to “Top dog goes in the house” or “First in last out” are helpful in reminding students that 1 ÷ 2 2\overline{1} 1/2 are all equivalent.

Order of Operations:

- Why do we need to specify order of operations?
- Can we find a “method” to the madness… why when calculating 3 + 5 · 2 would it make more (or less) sense to first compute 5 · 2 ?
- History: Ambiguity in order of operations can be seen throughout history, especially when operators have the same priority. In 1928 *A History of Mathematical Notations*, Cajori writes, "If an arithmetical or algebraic term contains ÷ and x, there is at present no agreement as to which sign shall be used first." Modern textbooks seem to agree that all multiplications and divisions should be performed in order from left to right.
- Please Excuse My Dear Aunt Sally is the most famous pneumonic in assisting with order of operations, although it should only be used as a guideline.

Other Operators:

- Power a^b
- Percent %
- Factorial !
**Functions as Operators:**
- Root \( \sqrt{ } \)
  
  Why is \( \sqrt{25} \) not ±5?
- Trigonometric functions \( \sin() \cos() \)
- Exponential function \( \exp() e^x \)
- Logarithmic function \( \log() \)

**Logical and Connective Operators:**
- And \( \land \)
- Or \( \lor \)
- Implies \( \Rightarrow \)
- If and only if \( \iff \)
- Therefore \( \therefore \)
- Because \( \because \)
- There exists \( \exists \)

**Relations:**
- Equality \( \equiv \sim \approx \cong \approx \equiv \) =
- Inequality \( \leq < \geq > \gg << \)

**Semantics:**
- Evaluate vs. simplify
  - You can **evaluate** a numerical expression by performing operations to obtain a single number or value.
  - You can **evaluate** an algebraic expression by finding the value by replacing each variable in an expression with numbers.
  - You can **simplify** an expression by using the rules of arithmetic and algebra to rewrite an expression as simply as possible.
- **Percent** vs. **percentage**
  - Both represent a proportion or share in relation to a whole (that whole is 100)
  - The word percent is an adverb and is defined as “out of 100, or per 100”
  - Example: *Forty percent of the alumni contributed to the endowment*.
  - The noun form of the word percent (or also called percentage) is also defined as “one part in 100.”
  - Example: *The hecklers constituted only a small percentage of the audience.*
- Square feet vs. feet squared
  - Recall the order of operations, and calculate \( 5 \cdot (1)^2 \) and \( (5 \cdot 1)^2 \)?
  - **Square feet** is a unit of area that is 1 foot by 1 foot.
    A square with a side of about 2.25 feet is equivalent to 5 square feet.
  - **Feet squared** refers to the length of a side of a square.
    5 feet squared is an area that measures 25 square feet.
  - It has to do with order of operations…
    \( x \text{ square feet} = x \cdot \text{feet}^2 \)
    \( x \text{ feet squared} = (x \cdot \text{feet})^2 \)