Mathematical Notations and all It Implies

**Purpose:** The purpose of this lab is to understand how to read mathematical notation, interpret symbols, convert math language into English and vice versa, and perhaps to appreciate the simplicity and beauty of mathematical notation.

**Outline:** In parts A, B and C, you are given a formal definition and will be expected to interpret it, and understand what it implies. In parts D and E you will be expected to show a few symbols not covered in class, and explain their meaning.

**Content Objectives:** Identify, interpret and evaluate mathematical symbols and notation

**Instructions:** Answer the following questions. Provide a cover page (date, project title, names)

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### Part A:
The formal definition of the absolute value function is given by

$$ |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}.$$  

1. Write the above definition in words (no math symbols or numbers)

2. Using the definition, show

$$ |x - a| = \begin{cases} x - a & x \geq a \\ a - x & x < a \end{cases}.$$  

3. Explain (in words) what the statement

$$ |x - a| > 0 \iff x \neq a$$

means

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### Part B:
The formal definition of a limit is given by

$$ \lim_{x \to a} f(x) = L \text{ if } \forall \varepsilon > 0 \exists \delta > 0 \text{ such that } 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon.$$  

1. Write the above definition in words (no math symbols or numbers)

2. The informal definition of a limit is as follows: The limit of $f(x)$ as $x$ approaches $a$ equals $L$ if we can make the values of $f(x)$ arbitrarily close to $L$ by taking $x$ to be sufficiently close to $a$ on either side (but not equivalent to $a$).

   Match the formal to the informal...
   
   – what part of the formal definition indicates “$f(x)$ arbitrarily close to $L$”
   
   – what part of the formal definition indicates “$x$ sufficiently close to $a$ on either side”
   
   – how does the formal definition insure that $x$ is not equivalent to $a$?

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### Part C:
A sequence of numbers is given by $a_1 = 1000, \ a_n = 1.03(a_{n-1}) + 100$

1. To find $a_2$ we use the formula $a_2 = 1.03(a_{1}) + 100 = 1.03(1,000) + 100 = 1,130$. Find $a_3, a_4, a_5$... up to $a_{10}$

2. What is the starting point for this sequence? Will it continue to get larger as $n$ gets larger?

3. Think of this in terms of an investment, where $a_n$ represents the amount of money in an investment at year $n$. What is the initial investment? What does the “$1.03(a_{n-1})$” represent? How about the “$+ 100$”?
Part D: There are many mathematical symbols we have not covered. Provide 3 examples. Include what they mean and in what context they are usually used.

Part E: Provide an example of a mathematical statement, expression or formula that you have encountered and interpret it.